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The properties of the electromagnetic field around the excited hydrogen atom—a quantum field theory approach

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Abstract. The properties of the electromagnetic field around a hydrogen atom in the excited $2p_{3/2}$ state are studied in relativistic quantum electrodynamics. The field theory definition of an excited state is introduced. This definition is used to find the time-dependent energy density and angular momentum density associated with the virtual photon cloud surrounding the atom as well as with real wave absorbed by the atom and emitted by it.

1. Introduction

The electromagnetic field surrounding the hydrogen atom—an object which consists of charged particles—has been investigated in the past from different points of view. Lately, particular attention is being paid to the, so called, virtual photon cloud in the case when atom is in the ground state (Compagno *et al* 1983, 1987, Passante *et al* 1985, Passante and Power 1987, Persico and Power 1986). In this situation the real wave cannot be emitted and only virtual processes, originating from quantum nature of the source, come into play. The spatial properties of the cloud have mainly been investigated in the language of quantum optics. In the preceding paper (Radożycki 1990) we have applied to this problem the full relativistic quantum electrodynamics. We have calculated the two characteristics of the cloud: the electromagnetic energy density distribution $\langle\frac{1}{2}(\boldsymbol{E}(\boldsymbol{x})^2+\boldsymbol{B}(\boldsymbol{x})^2)\rangle$ and the angular momentum density distribution $\langle\boldsymbol{x}\times(\boldsymbol{E}(\boldsymbol{x})\times\boldsymbol{B}(\boldsymbol{x}))\rangle$ in the space around the atom. Here we would like to develop the methods of this approach and to apply them to the situation in which the atom is in an excited state. The situation with the excited state is more interesting as we have to deal now with a dynamical problem. At the same time, however, it is much more complicated. All higher atomic states in quantum electrodynamics become unstable on account of interaction with the fluctuating electromagnetic field and, in this connection, they are not eigenstates of the new, *full* Hamiltonian. Thus the problem of the definition of the excited state arises. We deal with this interesting question in section 2. The states, which are usually used as unstable states in such a situation—the eigenstates of the Hamiltonian with the interaction term switched off—are not correct, physical, dressed states of the *system*. We postulate, therefore, a certain definition of the excited states. Besides, we assume that the full electron propagator in the Coulomb potential exhibits, for complex values of the energy, a pole corresponding to the resonance, lying on the unphysical sheet of the Riemann surface. Now, we cannot apply ordinary perturbation methods because we would lose time dependences of the type $e^{-\Gamma t}$ (the exponential would be expanded too). We will, therefore, make use of the Dyson-Schwinger equations for the appropriate Green functions omitting all the corrections to the vertices or vacuum polarization type for the external photon legs, but still retaining full electron propagators which have appropriate resonance poles. In section 3, equipped with the proper definition, we set about evaluating the spatial distribution and the time evolution of the cloud surrounding the atom in the excited $2p_{3/2}$ state. For that we have to develop a different method of calculation (different from that proposed in our work on the ground-state atom), because the expectation values of operators in a state, which is not the ground state and even an eigenstate of the Hamiltonian, cannot be easily transformed into the transition elements known from scattering theory. However, luckily, it was possible to derive an identity which allowed us to bypass these difficulties and to simplify lengthy calculations. In that way, in section 3, we get the distribution and the time dependence of the cloud around the physical excited state atom. Thanks to having used an appropriate definition of this state, no surface terms of the type δ or δ', connected with unphysical switching on of the interaction, arise (they are always present, when we deal with bare excited states).

The contribution to the energy density (and to the angular momentum density as well) arises not only from the virtual cloud, but also from the incoming and outgoing real wave. This contribution is considered in section 4.

2. The definition of the excited state

While setting about the investigation of the virtual cloud for the hydrogen atom in an excited state, we encounter a serious difficulty at the very beginning—the definition of a resonance state in quantum field theory, a state of a complicated coupled system: source and field. We do not have at our disposal any univocal criterion like, for instance, that of being the eigenstate of the total Hamiltonian in the case of the ground state. How to choose a proper definition then? In various physical problems where excited states come into play, one can proceed in the following way: switch off for a moment the interaction responsible for the decay of the unstable state and accept as its definition the corresponding eigenstate of the unperturbed, free Hamiltonian. In the case of an atom (a hydrogen atom for instance) one could take as an excited state the appropriate atomic eigenstate found in quantum mechanics. In this way, unfortunately, we fail to solve the problem we are dealing with. By turning off the interaction that causes an atom in an excited state to be unstable, we would simultaneously turn off the phenomena that we want to study. We would not have any virtual cloud round the atom at all! Both the decay and the formation of the cloud are consequences of the interaction of the electron bound in the atom with the quantized electromagnetic field. After having switched on the perturbation term, let us say at $t=0$, we would not observe anything until time $t=r/c$ where r is the distance to the observation point. Only after this time would the virtual cloud occur. (The dressing and undressing processes for the ground state were considered in Persico and Power (1987) and Compagno *et al* (1988a, b).) In the expression for the energy density we would also get terms of the type δ or δ' and higher derivatives of δ originating from the unphysical turning on of the electromagnetic coupling constant at $t=0$.

We are interested here in time evolution and not in transition amplitudes. Moreover, we are interested in the evolution for both short and long times. All this requires from us is a definition of an excited state which could describe *physics*, *dressed* states, such that may occur in reality. As we shall see, however, the bare state is not useless for us.

The calculation, at least partially, has to be performed 'non-perturbatively'—otherwise time dependence of the type $e^{-\Gamma t}$ would be lost.

Concerning the $2p_{3/2}$ state, we assume that the full electron propagator has a pole, in variable E, corresponding to this resonance, lying on the unphysical sheet of the Riemann surface (Møller 1946, Peierls 1955, Levy 1959, Eden *et al* 1966). Let us write the full, renormalized propagator S in the form

$$\begin{aligned} S_{\alpha\beta}(\tau, \boldsymbol{x}; 0, \boldsymbol{y}) &\\ &= \langle 0|\mathrm{T}(\Psi_\alpha(\tau, \boldsymbol{x})\bar{\Psi}_\beta(0, \boldsymbol{y}))|0\rangle \\ &= -\mathrm{i}\Theta(\tau)\langle 0|\Psi_\alpha(\tau, \boldsymbol{x})\bar{\Psi}_\beta(0, \boldsymbol{y})|0\rangle + \mathrm{i}\Theta(-\tau)\langle 0|\bar{\Psi}_\beta(0, \boldsymbol{y})\Psi_\alpha(\tau, \boldsymbol{x})|0\rangle. \end{aligned} \tag{1}$$

Only the first term is essential for our considerations since the second one cannot have a suitable ($2p_{3/2}$) singularity (Weldon 1976). Let us take the Fourier transform (over τ) of this expression.

$$S_E(\boldsymbol{x}, \boldsymbol{y}) = \left\langle 0 \left| \Psi(\boldsymbol{x}) \frac{1}{E + \mathrm{i}\varepsilon - H} \bar{\Psi}(\boldsymbol{y}) \right| 0 \right\rangle + \dots \tag{2}$$

The situation we have to deal with here is shown on figure 1. The expression (2) has a cut from E_1 (i.e. E_{1s}) to infinity. Expression (2) is found for E on the upper-plane of the physical sheet ($E + \mathrm{i}\varepsilon$). If we now make the analytical continuation to E lying in the lower half-plane, we reach for $E > E_1$ the unphysical sheet where the mentioned pole is situated (for complex energy $E = E_2 - \mathrm{i}\Gamma/2$). In order not to complicate the picture, we will forget about the $2p_{1/2}$ and $2s_{1/2}$ states lying on the energy scale below $2p_{3/2}$ but which are of little importance to us. We will consider exclusively the direct transitions to the ground state.

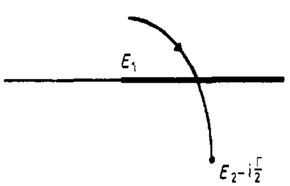

Figure 1. The E-plane for the Fourier transform of the propagator S.

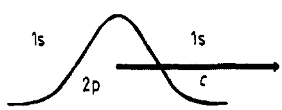

Figure 2. The time behaviour of the virtual cloud.

The electron propagator (2) has much more complicated behaviour in E than a simple pole. Some of the non-polar terms vanish when we perform the integrals over $\boldsymbol{x}$ and $\boldsymbol{y}$ with the wavefunctions of the $2p_{3/2}$ state which select from Ψ and $\bar{\Psi}$ pieces of appropriate quantum numbers.

$$\int \mathrm{d}^3x\, \mathrm{d}^3y\, \Psi_2^+(\boldsymbol{x}) \left\langle 0 \left| \Psi(\boldsymbol{x}) \frac{1}{E + \mathrm{i}\varepsilon - H} \bar{\Psi}(\boldsymbol{y}) \right| 0 \right\rangle \gamma^0 \Psi_2(\boldsymbol{y}).$$

In the calculations that constitute the content of the following section and of appendix 3, the confluence of resonance denominators plays an essential role. This suggests that we confine ourselves to the pole approximation for the propagator S:

$$\int d^3x\, d^3y\, \Psi_2^+(x)\left\langle 0\left|\Psi(x)\frac{1}{E+i\varepsilon-H}\bar{\Psi}(y)\right|0\right\rangle\gamma^0\Psi_2(y)\simeq\frac{1}{E-E_2+i\Gamma/2} \tag{3}$$

where the Ψ_2 are wavefunctions and the Ψ are field operators.

The possible residue is unimportant; by analogy with a stable case we have put it equal to unity.

The aim of this work is not the investigation of the full electron propagator and the consequences of its behaviour for the time evolution, but an attempt to describe the virtual cloud around the atom in an excited state in the simplest case—when we can limit ourselves to the pole (3). It is the consistent, *field theoretical* definition of a *dressed* excited state that causes problems. The possible deviations from the time behaviour that we get here come from the corrections to our definition and may be considered in the future as a further step.

The formula (3) suggests that we should try to define as a first step the state $|2'\rangle$ for which the resonance pole (3) occurs:

$$|2'\rangle=\int d^3x\,\bar{\Psi}(x)|0\rangle\gamma^0\Psi_2(x). \tag{4}$$

As was the case for the ground state, it turns out that we may take this function from ordinary quantum mechanics. And one more remark concerning our notation: the $2p_{3/2}$ state is denoted in short by $|\mathbf{2}\rangle$, as in the formula (4), the prime being put in to remind us that it is not yet the state we are striving for. The physical ground state, as in the preceding paper, is denoted by $|\mathbf{1}\rangle$.

The operator $\int\Psi^+\Psi_2$ constitutes simply a certain creation operator acting on the vacuum (Ψ is the *full* electron field). As we show in appendix 3, the definition (4) does not give yet the proper state. It gives just the 'bare' $2p_{3/2}$ (at least from the point of view of the virtual cloud far from the atom) but the physical state should not, however, be too distant. If we tried to express the 'bare' state $sp_{3/2}$ through the physical, dressed one we would undoubtedly have:

$$|2'\rangle=|\mathbf{2}\rangle+\ldots. \tag{5}$$

The dotts stand for the total remainder which is difficult to write down. If we suppose that the physical $2p_{3/2}$ state is to be prepared by hitting the ground state atom with one photon, we may ask how the state $|2'\rangle$ looks in the same in-basis. (This is the question concerning, in fact, the dots in (5).) Certainly it resembles the atom in the ground state but together with, possibly, multiphoton states. Let us try, therefore, to take from $|2'\rangle$ only the part that is attainable exclusively from the one-photon in states:

$$|\mathbf{2}\rangle=\sum\!\!\!\!\!\!\int_{k,\lambda}|1s,(\boldsymbol{k},\lambda)\text{in}\rangle\langle 1s,(\boldsymbol{k},\lambda)\text{in}|2'\rangle. \tag{6}$$

It does not mean, however, that the state $|\mathbf{2}\rangle$ decomposes simultaneously into only one-photon *out* states. If we would like to force it, this would immediately result in complex average values of certain physical operators.

The above argument leading to (6) is not precise, but as it will turn out in the following section, the state $|2'\rangle$ with the projector of (6) actually becomes the 'dressed' state. The above formula will constitute the definition of the physical, excited state we have sought. This is a postulate which may be verified only on the grounds of the results that can be obtained with the use of (6). As it seems to us, those obtained in the following sections speak in its favour.

In appendix 1 we calculate the wavepackets that display the manner in which the state $|2'\rangle$ is decomposed into the one-photon in and out states. These wavepackets will be useful in the following sections.

3. The virtual cloud properties

After the short discussion of the previous section concerning the definition of an excited state in quantum field theory, we can now, on the grounds of formula (6), set about calculating the quantity

$$\langle \mathbf{2}|\tfrac{1}{2}(\boldsymbol{E}(\boldsymbol{r},t)^2+\boldsymbol{B}(\boldsymbol{r},t)^2)|\mathbf{2}\rangle.$$

To this end we start with the quantity

$$\begin{aligned} I^{\mu\nu}(x,y) &= \langle \mathbf{2}|\mathrm{T}(A^{\mu}(x)A^{\nu}(y))|\mathbf{2}\rangle \\ &= \sum_{k,\lambda}\!\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\!\int \sum_{n}\!\!\!\!\!\int \langle 2'|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|n\mathrm{out}\rangle \\ &\quad \times\langle n\mathrm{out}|\mathrm{T}(A^{\mu}(x)A^{\nu}(y))|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|2'\rangle. \end{aligned} \tag{7}$$

By appropriate differentiations we can get from this object any combination of the fields E and B. In (7) we consider only the quantum part of the electromagnetic field. The classical part coming from the proton also gives its contribution to the full energy density. As it is, however, very simple and not very interesting, we will not write it here explicitly, taking it into account only in the final formulae.

Expressions similar to (7) arise also in appendix 3, where we find the magnetic field around the atom. We calculate there all the Feynman diagrams contributing to it to the lowest order in e. Here the number of the diagrams contributing to, for instance, the energy density is much bigger and therefore we have to proceed another way. Our calculation will be performed in stages. First of all we will divide $I^{\mu\nu}$ into two parts: A and B.

3.1. *Part A of $I^{\mu\nu}$*

Here we exclude the diagrams in which one (or both) of the fields A^{μ}, A^{ν} is 'joined' with one (or both) of the photons q and k (i.e. we do not consider in item A the energy density connected with the real wave that excites the atom or is emitted during decay). These are taken into account in B. Also we will now consider only the diagrams where A^{μ} is attached to the electron line to the left of A^{ν}. The symmetry $\{\mu\leftrightarrow\nu, x\leftrightarrow y\}$ will be included at the very end. The expression we are talking about in this subsection breaks up into three pieces.

(i) A^{μ} and A^{ν} are coupled to the electron line to the right of $|n\mathrm{out}\rangle$ (i.e. the diagram corresponding to $\langle n\mathrm{out}|\mathrm{T}(A^{\mu}(x)A^{\nu}(y))|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle$ is connected from the point of view of the A^{μ}, A^{ν} end electron).

(ii) A^{ν} couples to the right of $|n\mathrm{out}\rangle$ and A^{μ} to the left.

(iii) Both A^{μ} and A^{ν} are coupled to the left of $|n\mathrm{out}\rangle$.

Now let us consider these three cases one by one.

3.1.1. To order e^2 the contribution comes exclusively from the one-photon states $|n\mathrm{out}\rangle$. Skipping the steps similar to those given in appendix 3, we get

$$
\begin{aligned}
&\langle 1\mathrm{s}, (\boldsymbol{p}, \sigma)\mathrm{out}|\mathrm{T}(A^\mu(x)A^\nu(y))|1\mathrm{s}, (\boldsymbol{q}, \rho)\mathrm{in}\rangle^{\mathrm{A}_1}_{\mathrm{NS}} \\
&= \mathrm{i}e^4\bigg(\int \mathrm{d}^3w_1\,\mathrm{d}^4w_2\,\mathrm{d}^4w_3\,\mathrm{d}^3w_4\,\bar{\Psi}_1(\boldsymbol{w}_1)\,\mathrm{e}^{-\mathrm{i}\boldsymbol{p}\boldsymbol{w}_1}(\varepsilon_{\boldsymbol{p}}^{(\sigma)}\gamma)\Psi_2(\boldsymbol{w}_1) \\
&\times\frac{\exp[\mathrm{i}(p+E_1)w_2^0]}{p+E_1-E_2+\mathrm{i}(\Gamma/2)}\bar{\Psi}_2(w_2)\gamma^\mu S_v^{\mathrm{F}}(w_2, w_3)\gamma^\nu\Psi_2(w_3) \\
&\times\frac{\exp[-\mathrm{i}(q+E_1)w_3^0]}{q+E_1-E_2+\mathrm{i}(\Gamma/2)}\bar{\Psi}_2(\boldsymbol{w}_4)\,\mathrm{e}^{\mathrm{i}\boldsymbol{q}\boldsymbol{w}_4}(\varepsilon_{\boldsymbol{q}}^{(\rho)*}\gamma)\Psi_1(\boldsymbol{w}_4)\Delta^{\mathrm{F}}(x-w_2)\Delta^{\mathrm{F}}(y-w_3) \\
&-\mathrm{i}\int \mathrm{d}^3w_1\,\mathrm{d}^4w_2\,\mathrm{d}^4w_3\,\mathrm{d}^4w_4\,\bar{\Psi}_1(\boldsymbol{w}_1)\,\mathrm{e}^{-\mathrm{i}\boldsymbol{p}\boldsymbol{w}_1}(\varepsilon_{\boldsymbol{p}}^{(\sigma)}\gamma)\Psi_2(\boldsymbol{w}_1) \\
&\times\frac{\exp[\mathrm{i}(p+E_1-q-E_1)w_2^0]}{p+E_1-E_2+\mathrm{i}(\Gamma/2)} \\
&\times\Theta(w_2^0-w_3^0)\bar{\Psi}_2(\boldsymbol{w}_2)\,\mathrm{e}^{\mathrm{i}\boldsymbol{q}\boldsymbol{w}_2}(\varepsilon_{\boldsymbol{q}}^{(\rho)*}\gamma)\Psi_1(\boldsymbol{w}_2)\bar{\Psi}_1(w_3) \\
&\times\gamma^\mu S_v^{\mathrm{F}}(w_3, w_4)\gamma^\nu\Psi_1(w_4)\Delta^{\mathrm{F}}(x-w_3)\Delta^{\mathrm{F}}(y-w_4) \\
&-\mathrm{i}\int \mathrm{d}^4w_1\,\mathrm{d}^4w_2\,\mathrm{d}^4w_3\,\mathrm{d}^3w_4\,\bar{\Psi}_1(w_1)\gamma^\mu S_v^{\mathrm{F}}(w_1, w_2)\gamma^\nu\Psi_1(w_2) \\
&\times\Theta(w_2^0-w_3^0)\bar{\Psi}_1(\boldsymbol{w}_3)\,\mathrm{e}^{-\mathrm{i}\boldsymbol{p}\boldsymbol{w}_3}(\varepsilon_{\boldsymbol{p}}^{(\sigma)}\gamma)\Psi_2(\boldsymbol{w}_3)\bar{\Psi}_2(\boldsymbol{w}_4) \\
&\times\mathrm{e}^{\mathrm{i}\boldsymbol{q}\boldsymbol{w}_4}(\varepsilon_{\boldsymbol{q}}^{(\rho)*}\gamma)\Psi_1(\boldsymbol{w}_4)\,\frac{\exp[\mathrm{i}(p+E_1-q-E_1)w_3^0]}{q+E_1-E_2+\mathrm{i}(\Gamma/2)}\,\Delta^{\mathrm{F}}(x-w_1)\Delta^{\mathrm{F}}(y-w_2)\bigg) \\
&+\delta_\Gamma(\boldsymbol{p}-\boldsymbol{q})\delta_{\sigma\rho}\langle 1\mathrm{s}, 0\mathrm{out}|T(A^\mu(x)A^\nu(y))|1\mathrm{s}, 0\mathrm{in}\rangle. \qquad (8)
\end{aligned}
$$

The expression $\langle\mathbf{2}|\mathrm{T}(A^\mu(x)A^\nu(y))|\mathbf{2}\rangle_{\mathrm{NS}}$ (without the symmetry $\{\mu\leftrightarrow\nu, x\leftrightarrow y\}$) will be now called $J^{\mu\nu}(x, y)$. We have then

$$
\begin{aligned}
J^{\mu\nu}_{\mathrm{A}_1}(x, y) &= \sum_{k,\lambda}\!\!\!\!\int\;\sum_{p,\sigma}\!\!\!\!\int \frac{e\int \mathrm{d}^3w\,\bar{\Psi}_2(\boldsymbol{w})\,\mathrm{e}^{\mathrm{i}\boldsymbol{p}\boldsymbol{w}}(\varepsilon_{\boldsymbol{p}}^{(\sigma)*}\gamma)\Psi_1(\boldsymbol{w})}{p+E_1-E_2-\mathrm{i}(\Gamma/2)} \\
&\times\langle 1\mathrm{s}, (\boldsymbol{p}, \sigma)\mathrm{out}|\mathrm{T}(A^\mu(x)A^\nu(y))|1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in}\rangle_{\mathrm{NS}} \\
&\times\frac{e\int \mathrm{d}^3u\,\bar{\Psi}_1(\boldsymbol{u})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{u}}(\varepsilon_{\boldsymbol{k}}^{(\lambda)}\gamma)\Psi_2(\boldsymbol{u})}{k+E_1-E_2-\mathrm{i}(\Gamma/2)}. \qquad (9)
\end{aligned}
$$

We introduce here what we have got in (8) and make use of (A2.2). After these manipulations we have

$$
\begin{aligned}
J^{\mu\nu}_{\mathrm{A}_1}(x, y) &= \mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\,[\![\{\Theta(w_0)\exp[-(\Gamma/2)w_0]+\Theta(-w_0)\exp[(\Gamma/2)w_0]\} \\
&\times\bar{\Psi}_2(w)\gamma^\mu S_v^{\mathrm{F}}(w, z)\gamma^\nu\Psi_2(z)(\Theta(z_0)\exp[-(\Gamma/2)z_0]+\Theta(-z_0)\exp[(\Gamma/2)z_0]) \\
&+\Theta(-w_0)[1-\exp(\Gamma w_0)]\bar{\Psi}_1(w)\gamma^\mu S_v^{\mathrm{F}}(w, z)\gamma^\nu\Psi_1(z) \\
&+\bar{\Psi}_1(w)\gamma^\mu S_v^{\mathrm{F}}(w, z)\gamma^\nu\Psi_1(z)\Theta(z_0)[1-\exp(-\Gamma z_0)]]\!] \\
&\times\Delta^{\mathrm{F}}(x-w)\Delta^{\mathrm{F}}(y-z). \qquad (10)
\end{aligned}
$$

3.1.2. Here the important contribution is given not only by one-photon states. Without going into details as to which other out states should be taken into account in the sum, we can always write

$$J^{\mu\nu}_{A_2}(x,y)=\sum_{k,\lambda}\!\!\!\!\int\sum_{q,\rho}\!\!\!\!\int\sum_{p,\sigma}\!\!\!\!\int\sum_{n}\!\!\!\!\int \langle 2'|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\int \mathrm{d}^4z\,\Delta^{\mathrm{F}}(y-z)\partial_\alpha\partial^\alpha_z\,\mathrm{e}^{\mathrm{i}px}\varepsilon^{\mu(\sigma)}_p$$
$$\times\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|a^{(\sigma)\dagger}_{p\mathrm{out}}|n-\gamma\mathrm{out}\rangle\langle n-\gamma\mathrm{out}|A^\nu(z)|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle$$
$$\times\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle. \tag{11}$$

The symbol $|n-\gamma\rangle$ means that from the state $|n\rangle$ one photon has been taken off, or, if there were no photons at all, it gives no contribution to the sum. The d'Alambert operator $\partial_\alpha\partial^\alpha_z$ ensures that A^ν is not coupled to any photon present in $|n\mathrm{out}\rangle$ (the d'Alambert operator annihilates such expressions). Those contributions are considered in 3.1.3.

The states $|n-\gamma\mathrm{out}\rangle$ once again form a complete set, so that the sum may be easily performed:

$$J^{\mu\nu}_{A_2}(x,y)=\int \mathrm{d}^4z\,\mathrm{d}^4w\,\Delta^-(x-w)\partial_\alpha\partial^\alpha_w\Delta^{\mathrm{F}}(y-z)\partial_\beta\partial^\beta_z\langle\mathbf{2}|A^\mu(z)A^\nu(w)|\mathbf{2}\rangle_{\mathrm{A}} \tag{12}$$

or

$$J^{\mu\nu}_{A_2}(x,y)=\int \mathrm{d}^4z\,\mathrm{d}^4w\,\Delta^-(x-w)\partial_\alpha\partial^\alpha_w\Delta^{\mathrm{F}}(y-z)\partial_\beta\partial^\beta_z$$
$$\times[\Theta(w_0-z_0)J^{\mu\nu}_{\mathrm{A}}(w,z)+\Theta(z_0-w_0)J^{\nu\mu *}_{\mathrm{A}}(z,w)]. \tag{13}$$

3.1.3. Here we move both A^μ and A^ν to the left through $|n\mathrm{out}\rangle$:

$$J^{\mu\nu}_{A_3}(x,y)=\sum_{k,\lambda}\!\!\!\!\int\sum_{q,\rho}\!\!\!\!\int\sum_{n}\!\!\!\!\int\langle 2'|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\int \mathrm{d}^4z\,\Delta^-(y-z)\partial_\alpha\partial^\alpha_z$$
$$\times\int \mathrm{d}^4w\,\Delta^-(x-w)\partial_\alpha\partial^\alpha_w\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|\tilde{\mathrm{T}}(A^\mu(w)A^\nu(z))|n-2\gamma\mathrm{out}\rangle$$
$$\times\langle n-2\gamma\mathrm{out}|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle$$
$$=\int \mathrm{d}^4z\,\Delta^-(y-z)\partial_\alpha\partial^\alpha_z\int \mathrm{d}^4w\,\Delta^-(x-w)\partial_\alpha\partial^\alpha_w J^{\nu\mu *}_{\mathrm{A}}(z,w). \tag{14}$$

If we gather all the terms together, we come to the following relation for $J^{\mu\nu}_{\mathrm{A}}(x,y)$:

$$J^{\mu\nu}_{\mathrm{A}}(x,y)=J^{\mu\nu}_{A_1}(x,y)+\int \mathrm{d}^4w\,\mathrm{d}^4z[\Delta^-(x-w)\partial_\alpha\partial^\alpha_w\Delta^{\mathrm{F}}(y-z)$$
$$\times\partial_\beta\partial^\beta_z(\Theta(w_0-z_0)J^{\mu\nu}_{\mathrm{A}}(w,z)+\Theta(z_0-w_0)J^{\nu\mu *}_{\mathrm{A}}(z,w))$$
$$+\Delta^-(x-w)\partial_\alpha\partial^\alpha_w\Delta^-(y-z)\partial_\beta\partial^\beta_z J^{\nu\mu *}_{\mathrm{A}}(z,w)]. \tag{15}$$

What we have got after all these manipulations? The formula (15) constitutes a certain identity which expresses $J^{\mu\nu}_{\mathrm{A}}(x,y)$ in terms of $J^{\mu\nu}_{A_1}(x,y)$ and of itself. Iterating (15) and making use of

$$\partial_\alpha\partial^\alpha_x\Delta^-(x)=0 \tag{16}$$

we obtain

$$J_{\mathrm{A}}^{\mu\nu}(x,y)=J_{\mathrm{A}_1}^{\mu\nu}(x,y)+\int \mathrm{d}^4w\,\mathrm{d}^4z[\Delta^-(x-w)\partial_\alpha\partial_w^\alpha\Delta^{\mathrm{F}}(y-z)$$
$$\times\partial_\beta\partial_z^\beta(\Theta(w_0-z_0)J_{\mathrm{A}_1}^{\mu\nu}(w,z)+\Theta(z_0-w_0)J_{\mathrm{A}_1}^{\nu\mu *}(z,w))$$
$$+\Delta^-(x-w)\partial_\alpha\partial_w^\alpha\Delta^-(y-z)\partial_\alpha\partial_z^\alpha J_{\mathrm{A}_1}^{\nu\mu *}(z,w)]. \tag{17}$$

Now we see that we do not have to calculate explicitly the terms with more than one photon in the out state. Having once performed the task of finding $J_{\mathrm{A}_1}^{\mu\nu}(x,y)$ we have immediately got the whole of $J_{\mathrm{A}}^{\mu\nu}(x,y)$! We are not forced to evaluate all the Feynman diagrams but only a subgroup of them. In that way we easily obtain

$$J_{\mathrm{A}}^{\mu\nu}(x,y)=\mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\{\Theta(w_0)\exp[-(\Gamma/2)w_0]+\Theta(-w_0)\exp[(\Gamma/2)w_0]\}$$
$$\times\bar{\Psi}_2(w)\gamma^\mu S_v^{\mathrm{F}}(w,z)\gamma^\nu\Psi_2(z)\{\Theta(z_0)\exp[-(\Gamma/2)z_0]+\Theta(-z_0)\exp[(\Gamma/2)z_0]\}$$
$$\times(\Delta^{\mathrm{F}}(x-w)\Delta^{\mathrm{F}}(y-z)+\Theta(w_0-z_0)\Delta^-(x-w)\Delta^{\mathrm{F}}(y-z))$$
$$-\mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\{\Theta(w_0)\exp[-(\Gamma/2)w_0]+\Theta(-w_0)\exp[(\Gamma/2)w_0]\}$$
$$\times\bar{\Psi}_2(w)\gamma^\mu\gamma^0 S_v^{\mathrm{F}\dagger}(w,z)\gamma^0\gamma^\nu\Psi_2(z)$$
$$\times\{\Theta(z_0)\exp[-(\Gamma/2)z_0]+\Theta(-z_0)\exp[(\Gamma/2)z_0]\}$$
$$\times(\Delta^-(x-w)\Delta^-(y-z)+\Theta(z_0-w_0)\Delta^-(x-w)\Delta^{\mathrm{F}}(y-z))$$
$$+\mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\,\Theta(-w_0)(1-\mathrm{e}^{\Gamma w_0})\bar{\Psi}_1(w)\gamma^\mu S_v^{\mathrm{F}}(w,z)\gamma^\nu\Psi_1(z)$$
$$\times(\Delta^{\mathrm{F}}(x-w)\Delta^{\mathrm{F}}(y-z)+\Theta(w_0-z_0)\Delta^-(x-w)\Delta^{\mathrm{F}}(y-z))$$
$$-\mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\,\Theta(-w_0)(1-\mathrm{e}^{\Gamma w_0})\bar{\Psi}_1(w)\gamma^\mu\gamma^0 S_v^{\mathrm{F}\dagger}(w,z)$$
$$\times\gamma^0\gamma^\nu\Psi_1(z)(\Delta^-(x-w)\Delta^-(y-z)+\Theta(z_0-w_0)\Delta^-(x-w)$$
$$\times\Delta^{\mathrm{F}}(y-z))+\mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\,\bar{\Psi}_1(w)\gamma^\mu S_v^{\mathrm{F}}(w,z)\gamma^\nu\Psi_1(z)\Theta(z_0)$$
$$\times(1-\mathrm{e}^{-\Gamma z_0})(\Delta^{\mathrm{F}}(x-w)\Delta^{\mathrm{F}}(y-z)+\Theta(w_0-z_0)\Delta^-(x-w)$$
$$\times\Delta^{\mathrm{F}}(y-z))-\mathrm{i}e^2\int \mathrm{d}^4w\,\mathrm{d}^4z\,\bar{\Psi}_1(w)\gamma^\mu\gamma^0 S_v^{\mathrm{F}\dagger}(w,z)\gamma^0\gamma^\nu\Psi_1(z)$$
$$\times\Theta(z_0)(1-\mathrm{e}^{-\Gamma z_0})(\Delta^-(x-w)\Delta^-(y-z)$$
$$+\Theta(z_0-w_0)\Delta^-(x-w)\Delta^{\mathrm{F}}(y-z)) \tag{18}$$

and of course

$$I_{\mathrm{A}}^{\mu\nu}(x,y)=J_{\mathrm{A}}^{\mu\nu}(x,y)+J_{\mathrm{A}}^{\nu\mu}(y,x).$$

To proceed further we observe that the whole expression (18) breaks up into some pieces that can be calculated separately. We will then carry out more detailed calculation only for one of them, for the other ones we will write down only final formulae. First we deal with the expression that describes the decay (and the excitation) of the 2p state (the first two terms in (18)) leaving for a moment the contribution from the 1s state (the last four terms). This will constitute the contents of item I. Additionally, if we use

$$S^{\mathrm{F}}_{v}(w, z) = -\mathrm{i}\sum_{n^+}\!\!\!\!\!\!\int \Theta(w_0 - z_0)\Psi^{(+)}_n(w)\bar{\Psi}^{(+)}_n(z) + \mathrm{i}\sum_{n^-}\!\!\!\!\!\!\int \Theta(z_0 - w_0)\Psi^{(-)}_n(w)\bar{\Psi}^{(-)}_n(z) \tag{19}$$

the expression $I^{\mu\nu}_{\mathrm{A_I}}$ divides, in turn, into the contributions from different intermediate states. We will consider separately the following cases:

(a) $E_n > E_2$
(b) $E_n = E_2$
(c) $0 < E_n < E_2$
(d) $E_n < 0$.

For each group of states the calculation is carried out in a different manner. We do not give the details here and say a few words only on the calculation in case (a). We have then:

$$\begin{aligned} I^{\mu\nu}_{\mathrm{A_{I_a}}}(x, y) = e^2 \int \mathrm{d}^4w\, \mathrm{d}^4z \sum_{\substack{n\\(E_n > E_2)}}\!\!\!\!\!\!\!\!\int \; &\{\Theta(w_0) \exp[-(\Gamma/2)w_0] + \Theta(-w_0) \exp[(\Gamma/2)w_0]\} \\ &\times \bar{\Psi}_2(w)\gamma^\mu S^{\mathrm{F}}_v(w, z)\gamma^\nu\Psi_2(z)\{\Theta(z_0) \exp[-(\Gamma/2)z_0] + \Theta(-z_0) \exp[(\Gamma/2)z_0]\} \\ &\times (\Theta(w_0 - z_0)\Delta^{\mathrm{R}}(x - w)\Delta^{\mathrm{F}}(y - z) + \Theta(z_0 - w_0)\Delta^{-}(x - w)\Delta^{\mathrm{R}}(y - z)) \\ &+ \{\mu \leftrightarrow \nu, x \leftrightarrow y\}. \end{aligned} \tag{20}$$

Now we can manipulate equation (20) using (A4.1) and (A4.4). The calculation is lengthy and we omit it here. The final formula is the following:

$$\begin{aligned} I^{\mu\nu}_{\mathrm{A_{I_a}}}(x, y) = \frac{-\mathrm{i}e^2}{32\pi^3} \sum_{\substack{n\\(E_n > E_2)}}\!\!\!\!\!\!\!\!\int \; &\int \frac{\mathrm{d}^3w\, \mathrm{d}^3z}{|\boldsymbol{x} - \boldsymbol{w}|\,|\boldsymbol{y} - \boldsymbol{z}|} \bar{\Psi}_2(\boldsymbol{w})\gamma^\mu\Psi_n(\boldsymbol{w})\bar{\Psi}_n(\boldsymbol{z})\gamma^\nu\Psi_2(\boldsymbol{z}) \\ &\times \int_0^\infty \frac{\mathrm{d}\omega}{\omega + E_n - E_2} [\{\Theta(x_0 - |\boldsymbol{x} - \boldsymbol{w}|) \exp[-\Gamma(x_0 - |\boldsymbol{x} - \boldsymbol{w}|)] \\ &+ \Theta(|\boldsymbol{x} - \boldsymbol{w}| - x_0) \exp[\Gamma(x_0 - |\boldsymbol{x} - \boldsymbol{w}|)]\} \exp[\mathrm{i}\omega(x_0 - |\boldsymbol{x} - \boldsymbol{w}|) \\ &\times \{\exp[\mathrm{i}\omega(|\boldsymbol{y} - \boldsymbol{z}| - y_0)] - \exp[-\mathrm{i}\omega(|\boldsymbol{y} - \boldsymbol{z}| + y_0)]\} \\ &+ \{\Theta(y_0 - |\boldsymbol{y} - \boldsymbol{z}|) \exp[-\Gamma(y_0 - |\boldsymbol{y} - \boldsymbol{z}|)] \\ &+ \Theta(|\boldsymbol{y} - \boldsymbol{z}| - y_0) \exp[\Gamma(y_0 - |\boldsymbol{y} - \boldsymbol{z}|)]\} \exp[-\mathrm{i}\omega(y_0 - |\boldsymbol{y} - \boldsymbol{z}|)] \\ &\times \{\exp[\mathrm{i}\omega(x_0 + |\boldsymbol{x} - \boldsymbol{w}|)] - \exp[\mathrm{i}\omega(x_0 - |\boldsymbol{x} - \boldsymbol{w}|)]\}] \\ &+ \{\mu \leftrightarrow \nu, x \leftrightarrow y\} \end{aligned} \tag{21}$$

where we have already omitted the higher-order terms.

To get E and B and to make the dipole approximation we will have to perform some differentiations over x_0, y_0, $\boldsymbol{x}$ and $\boldsymbol{y}$. When they act on the functions $\Theta(\;)$ or $\mathrm{e}^{\Gamma(\;)}$ they give zero or higher-order expressions. Therefore in the function describing the time evolution we can at once put $x_0 = y_0 = t$ and $|\boldsymbol{x} - \boldsymbol{w}| \approx |\boldsymbol{y} - \boldsymbol{z}| \approx r$

$$\begin{aligned} I^{\mu\nu}_{\mathrm{A_{I_a}}} = \frac{e^2}{16\pi^3} (\Theta(t - r)\, \mathrm{e}^{-\Gamma(t-r)} + \Theta(r - t)\, \mathrm{e}^{\Gamma(t-r)}) &\sum_{\substack{n\\(E_n > E_2)}}\!\!\!\!\!\!\!\!\int \; \int \frac{\mathrm{d}^3w\, \mathrm{d}^3z}{|\boldsymbol{x} - \boldsymbol{w}|\,|\boldsymbol{y} - \boldsymbol{z}|} \\ \times \bar{\Psi}_2(\boldsymbol{w})\gamma^\mu\Psi_n(\boldsymbol{w})\bar{\Psi}_n(\boldsymbol{z})\gamma^\nu\Psi_2(\boldsymbol{z}) &\int_0^\infty \frac{\mathrm{d}\omega}{\omega + E_n - E_2} \exp[\mathrm{i}\omega(x_0 - y_0)] \\ \times \sin[\omega(|\boldsymbol{x} - \boldsymbol{w}| + |\boldsymbol{y} - \boldsymbol{z}|)] &+ \{\mu \leftrightarrow \nu, x \leftrightarrow y\}. \end{aligned} \tag{22}$$

For the cases (b), (c) and (d) corresponding to other groups of the intermediate states in (19) the calculations are worked out in the same spirit.

$$I^{\mu\nu}_{\mathrm{A_{I_b}}}=\frac{e^2}{16\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\int\frac{\mathrm{d}^3w\,\mathrm{d}^3z}{|x-w||y-z|}$$
$$\times\bar{\Psi}_2(w)\gamma^\mu\Psi_2(w)\bar{\Psi}_2(z)\gamma^\nu\Psi_2(z). \tag{23}$$

In order to simplify the point (b), which otherwise becomes complicated, we have already used the symmetry $\{\mu\leftrightarrow\nu,\ x\leftrightarrow y\}$

$$I^{\mu\nu}_{\mathrm{A_{I_c}}}=-\frac{e^2}{16\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\sum_{\substack{n\\(0<E_n<E_2)}}\!\!\!\!\!\!\!\int\ \int\frac{\mathrm{d}^3w\,\mathrm{d}^3z}{|x-w||y-z|}$$
$$\times\bar{\Psi}_2(w)\gamma^\mu\Psi_n(w)\bar{\Psi}_n(z)\gamma^\nu\Psi_2(z)\int_0^\infty\frac{\mathrm{d}\omega}{\omega+E_2-E_n}\exp[\mathrm{i}\omega(y_0-x_0)]$$
$$\times\sin[\omega(|x-w|+|y-z|)]+\frac{e^2}{16\pi^2}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})$$
$$\times\sum_{\substack{n\\(0<E_n<E_2)}}\!\!\!\!\!\!\!\int\ \int\frac{\mathrm{d}^3w\,\mathrm{d}^3z}{|x-w||y-z|}\bar{\Psi}_2(w)\gamma^\mu\Psi_n(w)\bar{\Psi}_n(z)\gamma^\nu\Psi_2(z)$$
$$\times\exp[\mathrm{i}(E_2-E_n)(x_0-|x-w|-y_0-|y-z|)]+\{\mu\leftrightarrow\nu,\ x\leftrightarrow y\} \tag{24}$$

$$I^{\mu\nu}_{\mathrm{A_{I_d}}}=-\frac{e^2}{16\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\sum_{\substack{n\\(E_n<0)}}\!\!\!\!\!\!\!\int\ \int\frac{\mathrm{d}^3w\,\mathrm{d}^3z}{|x-w||y-z|}$$
$$\times\bar{\Psi}_2(w)\gamma^\mu\Psi_n(w)\bar{\Psi}_n(z)\gamma^\nu\Psi_2(z)\int_0^\infty\frac{\mathrm{d}\omega}{\omega+E_2-E_n}\exp[\mathrm{i}\omega(y_0-x_0)]$$
$$\times\sin[\omega(|x-w|+|y-z|)]+\{\mu\leftrightarrow\nu,\ x\leftrightarrow y\}. \tag{25}$$

The second term in (24) is a 'real wave'-type term and we will neglect it here and combine it into with $I^{\mu\nu}_{\mathrm{B}}$ of the next section. In the formulae for $I^{\mu\nu}_{\mathrm{A}}$ used henceforth, the term in question is already omitted.

Now we must carry out the same programme for the four last terms in (18) (item II). They describe the time-dependent contribution from the atomic 1s state before, and the decay to this state after the excitation. This time there are three cases we have to deal with:

(a) $E_n>E_1$
(b) $E_n=E_1$
(c) $E_n<0$.

In order not to lengthen too much the calculational part of this work we give only the results. In item (b) we present only the real part which we will be interested in and which is much simpler than the whole $I^{\mu\nu}_{\mathrm{A_{II_b}}}$:

$$I^{\mu\nu}_{\mathrm{A_{II_a}}}=\frac{e^2}{16\pi^3}(\Theta(t-r)(1-\mathrm{e}^{-\Gamma(t-r)})+\Theta(r-t)(1-\mathrm{e}^{\Gamma(t-r)}))\sum_{\substack{n\\(E_n>E_1)}}\!\!\!\!\!\!\!\int\ \int\frac{\mathrm{d}^3w\,\mathrm{d}^3z}{|x-w||y-z|}$$
$$\times\bar{\Psi}_1(w)\gamma^\mu\Psi_n(w)\bar{\Psi}_n(z)\gamma^\nu\Psi_1(z)\int_0^\infty\frac{\mathrm{d}\omega}{\omega+E_n-E_1}\exp[\mathrm{i}\omega(x_0-y_0)]$$
$$\times\sin[\omega(|x-w|+|y-z|)]+\{\mu\leftrightarrow\nu,\ x\leftrightarrow y\} \tag{26}$$

$$Re\, I^{\mu\nu}_{A_{II_b}} = \frac{e^2}{16\pi^2}(\Theta(t-r)(1-e^{-\Gamma(t-r)})+\Theta(r-t)(1-e^{\Gamma(t-r)}))\int \frac{d^3w\, d^3z}{|x-w||y-z|}$$
$$\times \bar{\Psi}_1(w)\gamma^\mu\Psi_1(w)\bar{\Psi}_1(z)\gamma^\nu\Psi_1(z) \tag{27}$$

$$I^{\mu\nu}_{A_{II_c}} = -\frac{e^2}{16\pi^3}(\Theta(t-r)(1-e^{-\Gamma(t-r)})+\Theta(r-t)(1-e^{\Gamma(t-r)}))\sum_{\substack{n\\(E_n<0)}}\!\!\!\!\!\!\int \int \frac{d^3w\, d^3z}{|x-w||y-z|}$$
$$\times \bar{\Psi}_1(w)\gamma^\mu\Psi_n(w)\bar{\Psi}_n(z)\gamma^\nu\Psi_1(z)\int_0^\infty \frac{d\omega}{\omega+E_1-E_n}\exp[i\omega(y_0-x_0)]$$
$$\times \sin[\omega(|x-w|+|y-z|)]+\{\mu\leftrightarrow\nu,\, x\leftrightarrow y\}. \tag{28}$$

Having in this manner completed all the pieces of the formula (18), we can now, by appropriate differentiations, easily get the energy density of the virtual cloud. In the dipole approximation and after taking into account the proton contribution to the energy density we obtain

$$\tfrac{1}{2}\langle \boldsymbol{E}(\boldsymbol{r},t)^2\rangle_{\rm virt}$$
$$= \frac{e^2}{32\pi^3}(\Theta(t-r)\, e^{-\Gamma(t-r)}+\Theta(r-t)\, e^{\Gamma(t-r)})\sum_n\!\!\!\!\!\!\int{}' \langle 2|x^i|n\rangle\langle n|x^k|2\rangle$$
$$\times\left[\left(\frac{2}{r^6}(\delta^{ik}+3\hat{r}^k\hat{r}^i)-\frac{2\omega_{n2}^2}{r^4}(3\delta^{ik}+\hat{r}^k\hat{r}^i)+\frac{2\omega_{n2}^4}{r^2}(\delta^{ik}-\hat{r}^k\hat{r}^i)\right)f(2\omega_{n2}r)\right.$$
$$\left.+\left(\frac{4\omega_{n2}}{r^5}(\delta^{ik}+3\hat{r}^k\hat{r}^i)-\frac{4\omega_{n2}^3}{r^3}(\delta^{ik}-\hat{r}^k\hat{r}^i)\right)g(2\omega_{n2}r)-\frac{\omega_{n2}^3}{r^3}(\delta^{ik}-\hat{r}^k\hat{r}^i)\right]$$
$$+\frac{e^2}{32\pi^3}(\Theta(t-r)(1-e^{-\Gamma(t-r)})+\Theta(r-t)(1-e^{\Gamma(t-r)}))\sum_n\!\!\!\!\!\!\int{}' \langle 1|x^i|n\rangle\langle n|x^k|1\rangle$$
$$\times\left[\left(\frac{2}{r^6}(\delta^{ik}+3\hat{r}^k\hat{r}^i)-\frac{2\omega_{n1}^2}{r^4}(3\delta^{ik}+\hat{r}^k\hat{r}^i)+\frac{2\omega_{n1}^4}{r^2}(\delta^{ik}-\hat{r}^k\hat{r}^i)\right)f(2\omega_{n1}r)\right.$$
$$\left.+\left(\frac{4\omega_{n1}}{r^5}(\delta^{ik}+3\hat{r}^k\hat{r}^i)-\frac{4\omega_{n1}^3}{r^3}(\delta^{ik}-\hat{r}^k\hat{r}^i)\right)g(2\omega_{n1}r)-\frac{\omega_{n1}^3}{r^3}(\delta^{ik}-\hat{r}^k\hat{r}^i)\right] \tag{29}$$

$$\tfrac{1}{2}\langle \boldsymbol{B}(\boldsymbol{r},t)^2\rangle_{\rm virt}$$
$$= \frac{e^2}{16\pi^3}(\Theta(t-r)\, e^{-\Gamma(t-r)}+\Theta(r-t)\, e^{\Gamma(t-r)})\sum_n\!\!\!\!\!\!\int{}' \langle 2|x^k|n\rangle\langle n|x^i|2\rangle$$
$$\times\left[\left(\frac{\omega_{n2}^2}{r^4}-\frac{\omega_{n2}^4}{r^2}\right)f(2\omega_{n2}r)+\frac{2\omega_{n2}^3}{r^3}g(2\omega_{n2}r)+\frac{\omega_{n2}^3}{2r^3}\right](\delta^{ik}-\hat{r}^k\hat{r}^i)$$
$$+\frac{e^2}{16\pi^3}[\Theta(t-r)(1-e^{-\Gamma(t-r)})+\Theta(r-t)(1-e^{\Gamma(t-r)})]$$
$$\times\sum_n\!\!\!\!\!\!\int{}' \langle 1|x^k|n\rangle\langle n|x^i|1\rangle\left[\left(\frac{\omega_{n1}^2}{r^4}-\frac{\omega_{n1}^4}{r^2}\right)f(2\omega_{n1}r)\right.$$
$$\left.+\frac{2\omega_{n1}^3}{r^3}g(2\omega_{n1}r)+\frac{\omega_{n1}^3}{2r^3}\right](\delta^{ik}-\hat{r}^k\hat{r}^i) \tag{30}$$

where

$$\omega_{nm} = |E_n - E_m|. \tag{31}$$

The well known (Abramowitz and Stegun 1964) functions $f(z)$ and $g(z)$ are expressed as follows:

$$f(z) = \mathrm{ci}(z)\sin(z) - \mathrm{si}(z)\cos(z) \tag{32}$$

$$g(z) = -\mathrm{ci}(z)\cos(z) - \mathrm{si}(z)\sin(z) \tag{33}$$

where we use the definitions

$$\mathrm{si}(z) = \int_\infty^z \frac{\sin(t)}{t}\,\mathrm{d}t \tag{34}$$

$$\mathrm{ci}(z) = \int_\infty^z \frac{\cos(t)}{t}\,\mathrm{d}t \tag{35}$$

$$\sum_n\!\!\!\!\!\!\int{}' \langle m|r^i|n\rangle\langle n|r^k|m\rangle = \sum_{\substack{n\\ E_n>E_m}}\!\!\!\!\!\!\int \langle m|r^i|n\rangle\langle n|r^k|m\rangle - \sum_{\substack{n\\ E_n<E_m}}\!\!\!\!\!\!\int \langle m|r^i|n\rangle\langle n|r^k|m\rangle. \tag{36}$$

We have omitted in (30) the square of (A3.17)—the magnetic moment contribution to the energy which has nothing to do with the virtual cloud.

As we see, the state is already dressed—the time evolution shows that at no moment the virtual cloud disappears. This constitutes a kind of the verification of (6).

To complete the results we also give the expression for the angular momentum distribution:

$$\begin{aligned}
&\langle (\boldsymbol{r}\times(\boldsymbol{E}(\boldsymbol{r})\times\boldsymbol{B}(\boldsymbol{r})))_j\rangle_{\mathrm{virt}}\\
&\quad= -\frac{\mathrm{i}e^2}{16\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)} + \Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\\
&\qquad\times \sum_n\!\!\!\!\!\!\int{}' (\langle 2|x^k|n\rangle\langle n|x^l|2\rangle + \langle 2|x^l|n\rangle\langle n|x^k|2\rangle)\\
&\qquad\times\left[\omega_{n2}\left(\frac{2}{r^5} - \frac{4\omega_{n2}^2}{r^3}\right) f(2\omega_{n2}r) + \omega_{n2}\left(\frac{4}{r^4} - \frac{2\omega_{n2}^2}{r^2}\right)\right.\\
&\qquad\left.\times g(2\omega_{n2}r) + \frac{5\omega_{n2}}{2r^4}\right]\varepsilon_{jli}\frac{r^l r^k}{r} - \frac{\mathrm{i}e^2}{16\pi^3}\\
&\qquad\times[\Theta(t-r)(1-\mathrm{e}^{-\Gamma(t-r)}) + \Theta(r-t)(1-\mathrm{e}^{\Gamma(t-r)})]\\
&\qquad\times \sum_n\!\!\!\!\!\!\int{}' (\langle 1|x^k|n\rangle\langle n|x^l|1\rangle + \langle 1|x^l|n\rangle\langle n|x^k|1\rangle)\\
&\qquad\times\left[\omega_{n1}\left(\frac{2}{r^5} - \frac{4\omega_{n1}^2}{r^3}\right) f(2\omega_{n1}r) + \omega_{n1}\left(\frac{4}{r^4} - \frac{2\omega_{n1}^2}{r^2}\right)\right.\\
&\qquad\left.\times g(2\omega_{n1}r) + \frac{5\omega_{n1}}{2r^4}\right]\varepsilon_{jli}\frac{r^l r^k}{r}.
\end{aligned} \tag{37}$$

In the far (wave) zone approximation where we can make use of the well known behaviour of the functions f and g for large arguments (Abramowitz and Stegun 1964):

$$f(z)=\frac{1}{z}\left(1-\frac{2!}{z^2}+\frac{4!}{z^4}-\frac{6!}{z^6}+\dots\right) \tag{38}$$

$$g(z)=\frac{1}{z^2}\left(1-\frac{3!}{z^2}+\frac{5!}{z^4}-\frac{7!}{z^6}+\dots\right) \tag{39}$$

these formulae become

$$\begin{aligned}\tfrac{1}{2}\langle \boldsymbol{E}^2(\boldsymbol{r})\rangle_{\text{virt}}^{\text{FZ}}=&\frac{e^2}{64\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\\ &\times\sum_n\!\!\!\!\!\!\int\frac{1}{E_n-E_2}\langle 2|x^i|n\rangle\langle n|x^k|2\rangle\frac{1}{r^7}(13\delta_{ik}+7\hat{r}^i\hat{r}^k)\\ &+\frac{e^2}{64\pi^3}[\Theta(t-r)(1-\mathrm{e}^{-\Gamma(t-r)})+\Theta(r-t)(1-\mathrm{e}^{\Gamma(t-r)})]\\ &\times\sum_n\!\!\!\!\!\!\int\frac{1}{E_n-E_1}\langle 1|x^i|n\rangle\langle n|x^k|1\rangle\frac{1}{r^7}(13\delta_{ik}+7\hat{r}^i\hat{r}^k)\end{aligned} \tag{40}$$

$$\begin{aligned}\tfrac{1}{2}\langle \boldsymbol{B}^2(\boldsymbol{r})\rangle_{\text{virt}}^{\text{FZ}}=&-\frac{7e^2}{64\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\\ &\times\sum_n\!\!\!\!\!\!\int\frac{1}{E_n-E_2}\langle 2|x^i|n\rangle\langle n|x^k|2\rangle(\delta^{ik}-\hat{r}^i\hat{r}^k)\frac{1}{r^7}\\ &-\frac{7e^2}{64\pi^3}[\Theta(t-r)(1-\mathrm{e}^{-\Gamma(t-r)})+\Theta(r-t)(1-\mathrm{e}^{\Gamma(t-r)})]\\ &\times\sum_n\!\!\!\!\!\!\int\frac{1}{E_n-E_1}\langle 1|x^i|n\rangle\langle n|x^k|1\rangle(\delta^{ik}-\hat{r}^i\hat{r}^k)\frac{1}{r^7}\end{aligned} \tag{41}$$

$$\begin{aligned}&\langle(\boldsymbol{r}\times(\boldsymbol{E}(\boldsymbol{r})\times\boldsymbol{B}(\boldsymbol{r})))_j\rangle_{\text{virt}}^{\text{FZ}}\\ &\quad=\frac{35\mathrm{i}e^2}{16\pi^3}(\Theta(t-r)\,\mathrm{e}^{-\Gamma(t-r)}+\Theta(r-t)\,\mathrm{e}^{\Gamma(t-r)})\\ &\quad\times\sum_n\!\!\!\!\!\!\int\frac{1}{(E_n-E_2)^2}(\langle 2|x^k|n\rangle\langle n|x^i|2\rangle-\langle 2|x^i|n\rangle\langle n|x^k|2\rangle)\varepsilon_{jli}\frac{\hat{r}^l\hat{r}^k}{r^7}\\ &\quad+\frac{35\mathrm{i}e^2}{16\pi^3}[\Theta(t-r)(1-\mathrm{e}^{-\Gamma(t-r)})+\Theta(r-t)(1-\mathrm{e}^{\Gamma(t-r)})]\\ &\quad\times\sum_n\!\!\!\!\!\!\int\frac{1}{(E_n-E_1)^2}(\langle 1|x^k|n\rangle\langle n|x^i|1\rangle-\langle 1|x^i|n\rangle\langle n|x^k|1\rangle)\varepsilon_{jli}\frac{\hat{r}^l\hat{r}^k}{r^7}.\end{aligned} \tag{42}$$

The spatial distribution of the cloud is analogous to that obtained in our previous paper, the difference originating from the dissimilar geometry of 2p and 1s states. There are, therefore, some changes in the angular distribution since

$$\sum_n\!\!\!\!\!\!\int\frac{1}{(E_n-E_2)^\alpha}\langle 2|x^i|n\rangle\langle n|x^k|2\rangle\neq\sum_n\!\!\!\!\!\!\int\frac{1}{(E_n-E_1)^\alpha}\langle 1|x^i|n\rangle\langle n|x^k|1\rangle.$$

The principal feature of the formulae for the 2p state (29), (30), (37), (40), (41) and (42) is, however, their time dependence—the rebuilding of the cloud in the course of the photon absorption and emission process. The region in space where the cloud is such as is required by the geometry of the 2p state moves (in the radial direction) with the velocity of light. Its size is determined by the lifetime of the excited state. (The situation is shown in figure 2.) Outside, up to the exponential terms $e^{-\Gamma r}$, the distribution for the ground state does stand. At no moment is the distribution given in the whole of space by the atomic 2p state. This agrees with our expectations concerning the *physical* excited state. Only in the region sufficiently near to the atom can the cloud be readjusted before the atom starts to decay. For regions that are more distant the information about the excitation comes later.

The time evolution is symmetric. The process of a photon absorption looks (at least from the point of view of the virtual cloud) similar to the process of emission (but backward in time). This is due to the excitation accomplished by one photon. (The incoming wave is built up of one photon.)

4. The real wave properties

In this section we consider only those diagrams of (7) for which at least one of the fields A^μ and A^ν is attached to the photon k or q (item B). The rule that A^μ is always connected to the left of A^ν still holds good:

$$\begin{aligned}
&\langle \mathbf{2}|T(A^\mu(x)A^\nu(y))|\mathbf{2}\rangle^{\mathrm{B}}_{\mathrm{NS}} \\
&\quad=\sum_{k,\lambda}\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\int \sum_{n}\!\!\!\!\int \langle 2'|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|n\mathrm{out}\rangle_{\mathrm{B}} \\
&\qquad\times\langle n\mathrm{out}|\mathrm{T}(A^\mu(x)A^\nu(y))|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle^{\mathrm{B}}_{\mathrm{NS}}\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle \\
&\quad=\sum_{k,\lambda}\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\int \langle 2'|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|A^\mu(x)|1\mathrm{s},0\mathrm{in}\rangle \\
&\qquad\times\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle\varepsilon_{\boldsymbol{k}}^{\nu(\lambda)*}\,\mathrm{e}^{-\mathrm{i}ky}+\sum_{k,\lambda}\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\int \int \mathrm{d}^4z\,\Delta^{\mathrm{F}}(y-z)\partial_\alpha\partial_z^\alpha \\
&\qquad\times\langle 2'|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\varepsilon_{\boldsymbol{q}}^{\mu(\rho)}\,\mathrm{e}^{\mathrm{i}qx}\langle 1\mathrm{s},0\mathrm{out}|A^\nu(z)|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle \\
&\qquad\times\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle+\sum_{k,\lambda}\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\int \int \mathrm{d}^4z\,\Delta^{-}(y-z)\partial_\alpha\partial_z^\alpha\langle 2'|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\varepsilon_{\boldsymbol{q}}^{\mu(\rho)}\,\mathrm{e}^{\mathrm{i}qx} \\
&\qquad\times\langle 1\mathrm{s},0\mathrm{in}|A^\nu(z)|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle.
\end{aligned}\tag{43}$$

The calculation which can already be called our standard one gives

$$\begin{aligned}
&\sum_{q,\rho}\!\!\!\!\int \langle 1\mathrm{s},0\mathrm{out}|A^\mu(x)|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|2'\rangle \\
&\qquad=e\int \mathrm{d}^4w\,\Delta^{\mathrm{F}}(x-w)\bar{\Psi}_1(w)\gamma^\mu\Psi_2(w)(\Theta(w_0)\exp[-(\Gamma/2)w_0] \\
&\qquad\quad+\Theta(-w_0)\exp[(\Gamma/2)w_0])-e\int \mathrm{d}^4w\,\Delta^{-}(x-w) \\
&\qquad\quad\times\bar{\Psi}_1(w)\gamma^\mu\Psi_2(w)\Theta(-w_0)\exp[(\Gamma/2)w_0].
\end{aligned}\tag{44}$$

It is very easy to find all the terms of $J_{\mathrm{B}}^{\mu\nu}$, because $|1\mathrm{s}, 0\mathrm{in}\rangle = |1\mathrm{s}, 0\mathrm{out}\rangle$. Saving room, we give here only the final result:

$$J_{\mathrm{B}}^{\mu\nu}(x, y) = -e^2 \int \mathrm{d}^4 w\, \mathrm{d}^4 z\, \Theta(-w_0) \exp[(\Gamma/2) w_0] \bar{\Psi}_2(w) \gamma^\mu \Psi_1(w) \Delta^-(x-w)$$
$$\times \{\Theta(z_0) \exp[-(\Gamma/2) z_0] + \Theta(-z_0) \exp[(\Gamma/2) z_0]\}$$
$$\times \bar{\Psi}_1(z) \gamma^\nu \Psi_2(z) \Delta^{\mathrm{R}}(y-z) - e^2 \int \mathrm{d}^4 w\, \mathrm{d}^4 z$$
$$\times \{\Theta(w_0) \exp[-(\Gamma/2) w_0] + \Theta(-w_0) \exp[(\Gamma/2) w_0]\} \bar{\Psi}_2(w) \gamma^\mu \Psi_1(w)$$
$$\times \Delta^{\mathrm{R}}(x-w) \Theta(-z_0) \exp[(\Gamma/2) z_0] \bar{\Psi}_1(z) \gamma^\nu \Psi_2(z) \Delta^+(y-z)$$
$$+ e^2 \int \mathrm{d}^4 w\, \mathrm{d}^4 z\, \Theta(-w_0) \exp[(\Gamma/2) w_0] \bar{\Psi}_2(w) \gamma^\mu \Psi_1(w) \Delta^-(x-w)$$
$$\times \Theta(-z_0) \exp[(\Gamma/2) z_0] \bar{\Psi}_1(z) \gamma^\nu \Psi_2(z) \Delta^+(y-z). \tag{45}$$

If we take into account the remark on (24) we have

$$I_{\mathrm{B}}^{\mu\nu}(x, y) = e^2 \int \mathrm{d}^4 w\, \mathrm{d}^4 z [\![\{\Theta(w_0) \exp[-(\Gamma/2) w_0] + \Theta(-w_0) \exp[(\Gamma/2) w_0]\}$$
$$\times \{\Theta(z_0) \exp[-(\Gamma/2) z_0) + \Theta(-z_0) \exp[(\Gamma/2) z_0]\}$$
$$\times \Delta^{\mathrm{R}}(x-w) \Delta^{\mathrm{R}}(y-z) - \Theta(-w_0) \exp[(\Gamma/2) w_0]$$
$$\times \{\Theta(z_0) \exp[-(\Gamma/2) z_0] + \Theta(-z_0) \exp[(\Gamma/2) z_0]\} \Delta^-(x-w) \Delta^{\mathrm{R}}(y-z)$$
$$- \{\Theta(w_0) \exp[-(\Gamma/2) w_0] + \Theta(-w_0) \exp[(\Gamma/2) w_0]\} \Theta(-z_0)$$
$$\times \exp[(\Gamma/2) z_0] \Delta^{\mathrm{R}}(x-w) \Delta^+(y-z) + \Theta(-w_0) \exp[(\Gamma/2) w_0]$$
$$\times \Theta(-z_0) \exp[(\Gamma/2) z_0] \Delta^-(x-w) \Delta^+(y-z)]\!] \bar{\Psi}_2(w) \gamma^\mu \Psi_1(w)$$
$$\times \bar{\Psi}_1(z) \gamma^\nu \Psi_2(z) + \{\mu \leftrightarrow \nu, x \leftrightarrow y\}. \tag{46}$$

Below we will calculate the individual parts of (46)

$$I_{\mathrm{B_I}}^{\mu\nu}(x, y) = e^2 \int \mathrm{d}^4 w\, \mathrm{d}^4 z\, \Theta(w_0) \exp[-(\Gamma/2) w_0] \Theta(z_0) \exp[-(\Gamma/2) z_0] \bar{\Psi}_2(w) \gamma^\mu \Psi_1(w)$$
$$\times \bar{\Psi}_1(z) \gamma^\nu \Psi_2(z) \Delta^{\mathrm{R}}(x-w) \Delta^{\mathrm{R}}(y-z) + \{\mu \leftrightarrow \nu, x \leftrightarrow y\}$$
$$= \frac{e^2}{16\pi^3} \int \frac{\mathrm{d}^3 w\, \mathrm{d}^3 z}{|\boldsymbol{x}-\boldsymbol{w}||\boldsymbol{y}-\boldsymbol{z}|} \bar{\Psi}_2(\boldsymbol{w}) \gamma^\mu \Psi_1(\boldsymbol{w}) \bar{\Psi}_1(\boldsymbol{z}) \gamma^\nu \Psi_2(\boldsymbol{z})$$
$$\times \Theta(x_0 - |\boldsymbol{x}-\boldsymbol{w}|) \exp[\mathrm{i}(E_2 - E_1 + \mathrm{i}\Gamma/2)(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)] \Theta(y_0 - |\boldsymbol{y}-\boldsymbol{z}|)$$
$$\times \exp[-\mathrm{i}(E_2 - E_1 - \mathrm{i}\Gamma/2)(y_0 - |\boldsymbol{y}-\boldsymbol{z}|)] + \{\mu \leftrightarrow \nu, x \leftrightarrow y\} \tag{47}$$

$$I_{\mathrm{B_{II}}}^{\mu\nu}(x, y) = e^2 \int \mathrm{d}^4 w\, \mathrm{d}^4 z\, \Theta(w_0) \exp[-(\Gamma/2) w_0] \Theta(-z_0) \exp[(\Gamma/2) z_0] \bar{\Psi}_2(w) \gamma^\mu \Psi_1(w)$$
$$\times \bar{\Psi}_1(z) \gamma^\nu \Psi_2(z) (\Delta^{\mathrm{R}}(x-w) \Delta^{\mathrm{R}}(y-z) + \Delta^{\mathrm{R}}(x-w) \Delta^+(y-z)) + \{\mu \leftrightarrow \nu, x \leftrightarrow y\}$$
$$= \frac{e^2}{16\pi^3} \int \frac{\mathrm{d}^3 w\, \mathrm{d}^3 z}{|\boldsymbol{x}-\boldsymbol{w}||\boldsymbol{y}-\boldsymbol{z}|} \bar{\Psi}_2(\boldsymbol{w}) \gamma^\mu \Psi_1(\boldsymbol{w}) \bar{\Psi}_1(\boldsymbol{z}) \gamma^\nu \Psi_2(\boldsymbol{z})$$
$$\times \Theta(x_0 - |\boldsymbol{x}-\boldsymbol{w}|) \exp[\mathrm{i}(E_2 - E_1 + \mathrm{i}\Gamma/2)(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)]$$
$$\times \int_0^\infty \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\omega y_0} \sin(\omega |\boldsymbol{y}-\boldsymbol{z}|) \frac{1}{\omega + E_2 - E_1 + \mathrm{i}\Gamma/2} + \{\mu \leftrightarrow \nu, x \leftrightarrow y\} \tag{48}$$

$$I^{\mu\nu}_{B_{III}}(x, y) = e^2 \int d^4w\, d^4z\, \Theta(-w_0) \exp[(\Gamma/2)w_0]\Theta(z_0) \exp[-(\Gamma/2)z_0]\bar{\Psi}_2(w)\gamma^\mu\Psi_1(w)$$
$$\times \bar{\Psi}_1(z)\gamma^\nu\Psi_2(z)(\Delta^R(x-w)\Delta^R(y-z)$$
$$+\Delta^-(x-w)\Delta^R(y-z)) + \{\mu\leftrightarrow\nu, x\leftrightarrow y\}$$
$$= \frac{e^2}{16\pi^3} \int \frac{d^3w\, d^3z}{|\boldsymbol{x}-\boldsymbol{w}||\boldsymbol{y}-\boldsymbol{z}|} \bar{\Psi}_2(\boldsymbol{w})\gamma^\mu\Psi_1(\boldsymbol{w})\bar{\Psi}_1(\boldsymbol{z})\gamma^\nu\Psi_2(\boldsymbol{z})$$
$$\times \Theta(y_0 - |\boldsymbol{y}-\boldsymbol{z}|) \exp[i(E_1 - E_2 + i\Gamma/2)(y_0 - |\boldsymbol{y}-\boldsymbol{z}|)] \int_0^\infty d\omega\, e^{-i\omega x_0}$$
$$\times \sin(\omega|\boldsymbol{x}-\boldsymbol{w}|) \frac{1}{\omega + E_2 - E_1 - i\Gamma/2} + \{\mu\leftrightarrow\nu, x\leftrightarrow y\} \tag{49}$$

$$I^{\mu\nu}_{B_{IV}}(x, y) = e^2 \int d^4w\, d^4z\, \Theta(-w_0) \exp[(\Gamma/2)w_0]\Theta(-z_0) \exp[(\Gamma/2)z_0]\bar{\Psi}_2(w)\gamma^\mu\Psi_1(w)$$
$$\times \bar{\Psi}_1(z)\gamma^\nu\Psi_2(z)(\Delta^R(x-w)\Delta^R(y-z) - \Delta^-(x-w)\Delta^R(y-z)$$
$$-\Delta^R(x-w)\Delta^+(y-z) + \Delta^-(x-w)\Delta^+(y-z)) + \{\mu\leftrightarrow\nu, x\leftrightarrow y\}$$
$$= \frac{e^2}{64\pi^4} \int \frac{d^3w\, d^3z}{|\boldsymbol{x}-\boldsymbol{w}||\boldsymbol{y}-\boldsymbol{z}|} \bar{\Psi}_2(\boldsymbol{w})\gamma^\mu\Psi_1(\boldsymbol{w})\bar{\Psi}_1(\boldsymbol{z})\gamma^\nu\Psi_2(\boldsymbol{z})$$
$$\times \Big(2\pi\Theta(-x_0 - |\boldsymbol{x}-\boldsymbol{w}|) \exp[i(E_2 - E_1 - i\Gamma/2)(x_0 + |\boldsymbol{x}-\boldsymbol{w}|)]$$
$$+ 2\int_0^\infty \frac{d\omega_1}{\omega_1 + E_2 - E_1 - i\Gamma/2} e^{-i\omega_1 x_0} \sin(\omega_1|\boldsymbol{x}-\boldsymbol{w}|)\Big)$$
$$\times \Big(2\pi\Theta(-y_0 - |\boldsymbol{y}-\boldsymbol{z}|) \exp[i(E_1 - E_2 - i\Gamma/2)(y_0 + |\boldsymbol{y}-\boldsymbol{z}|)]$$
$$+ 2\int_0^\infty \frac{d\omega_2}{\omega_2 + E_2 - E_1 + i\Gamma/2} e^{i\omega_2 y_0} \sin(\omega_2|\boldsymbol{y}-\boldsymbol{z}|)\Big). \tag{50}$$

Lumping them together, performing integrals over the ω_i and neglecting the terms of order $e^2\Gamma/\Delta E$, we obtain

$$I^{\mu\nu}_{B}(x, y) = \frac{e^2}{16\pi^2} \int \frac{d^3w\, d^3z}{|\boldsymbol{x}-\boldsymbol{w}||\boldsymbol{y}-\boldsymbol{z}|} \bar{\Psi}_2(\boldsymbol{w})\gamma^\mu\Psi_1(\boldsymbol{w})\bar{\Psi}_1(\boldsymbol{z})\gamma^\nu\Psi_2(\boldsymbol{z})(h^*(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)$$
$$+ h(-x_0 - |\boldsymbol{x}-\boldsymbol{w}|))(h(y_0 - |\boldsymbol{y}-\boldsymbol{z}|) + h^*(-y_0 - |\boldsymbol{y}-\boldsymbol{z}|)) \tag{51}$$

where

$$h(z) = \exp[i(E_1 - E_2)z]\Big(\Theta(z) \exp[-(\Gamma/2)z] - \frac{i}{2\pi} \mathrm{E_i}(-i(E_1 - E_2)z)\Big) \tag{52}$$

and $\mathrm{E_i}$ is the integral exponent function.

If we now want to find the energy density contained in the incoming and outgoing waves we meet with some difficulty. The multipolar expansion is of no use close to the light cone. The setting of the atomic size equal to zero immediately leads to infinities. We have to proceed in a different way. We will investigate the energy density near the light cone but not on it, where we would get infinity (for $a_0 = 0$). The situation is shown on figure 3.

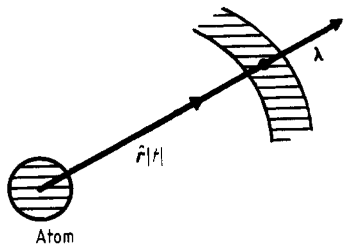

Figure 3. The variables describing the incoming and outgoing waves.

At time t the maximum of the wave is situated on the sphere of radius $|t|$. We introduce a new convenient variable in the following way:

$$\boldsymbol{r}=\begin{cases}\hat{\boldsymbol{r}}t+\boldsymbol{\lambda} & t>0 \qquad (53)\\ -\hat{\boldsymbol{r}}t+\boldsymbol{\lambda} & t<0. \qquad (54)\end{cases}$$

Since we are investigating the wave only in the radial direction, we can write

$$\boldsymbol{r}=(|t|+\lambda)\hat{\boldsymbol{r}}. \tag{55}$$

About λ, which will be now a scalar variable describing the shape of the packet, we assume

$$a_0 \ll \lambda \ll r.$$

Then the arguments of the function h become simply

$$t-|\boldsymbol{r}-\boldsymbol{w}|\approx\lambda \qquad t>0 \tag{56}$$

$$-t-|\boldsymbol{r}-\boldsymbol{w}|\approx\lambda \qquad t<0. \tag{57}$$

It is now very easy to obtain the formulae for the energy density:

$$\begin{aligned}\tfrac{1}{2}\langle \boldsymbol{E}^2\rangle_{\text{wave}}=&\frac{e^2}{16\pi^2}(E_2-E_1)^4\langle 2|x^i|1\rangle\langle 1|x^k|2\rangle\frac{1}{r^2}\\ &\times\left\{\delta^{ik}\left(h(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\left(h^*(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\right.\\ &+\hat{r}^i\hat{r}^k\left[-\left(h(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\left(h^*(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\right.\\ &\left.\left.+\frac{1}{4\pi^2(E_2-E_1)^4\lambda^4}\right]\right\}\end{aligned} \tag{58}$$

$$\begin{aligned}\tfrac{1}{2}\langle \boldsymbol{B}\rangle^2_{\text{wave}}=&\frac{e^2}{16\pi^2}(E_2-E_1)^4\langle 2|x^i|1\rangle\langle 1|x^k|2\rangle\frac{1}{r^2}\\ &\times(\delta^{ik}-\hat{r}^i\hat{r}^k)\left(h(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\left(h^*(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\end{aligned} \tag{59}$$

and for the angular momentum density:

$$\begin{aligned}\langle[\boldsymbol{r}\times(\boldsymbol{E}\times\boldsymbol{B})]_i\rangle_{\text{wave}}\\ =&\frac{ie^2}{32\pi^2}(E_2-E_1)^2\langle 2|x^i|1\rangle\langle 1|x^k|2\rangle\frac{1}{r^2}\left[\varepsilon_{ilj}\hat{r}^l\hat{r}^k\left(h(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\right.\\ &\left.-\varepsilon_{ilk}\hat{r}^l\hat{r}^j\left(h^*(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\right]\frac{1}{\lambda^2}.\end{aligned} \tag{60}$$

The expression $\langle 2|x^i|1\rangle\langle 1|x^k|2\rangle$ may be evaluated explicitly (and in consequence also the angular dependence of $\boldsymbol{E}^2$, $\boldsymbol{B}^2$ and $\boldsymbol{r}\times(\boldsymbol{E}\times\boldsymbol{B})$), if we use the known wavefunctions from quantum mechanics. It has the form

$$\langle 2|x^i|1\rangle\langle 1|x^k|2\rangle = N(\delta^{ik} - \hat{j}^i\hat{j}^k + i\hat{j}^l\varepsilon^{lik}) \tag{61}$$

where N is unessential constant. One can introduce it into (58)-(60), to get the angular dependences:

$$\tfrac{1}{2}\langle \boldsymbol{E}^2\rangle_{\text{wave}} = \frac{C}{t^2}(1+\cos^2\theta)\left(h(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\left(h^*(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)$$
$$+\frac{C}{r^2}(1-\cos^2\theta)\frac{1}{4\pi^2(E_2-E_1)^4\lambda^4} \tag{62}$$

$$\tfrac{1}{2}\langle \boldsymbol{B}^2\rangle_{\text{wave}} = \frac{C}{r^2}(1+\cos^2\theta)\left(h(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right)\left(h^*(\lambda)+\frac{1}{2\pi(E_2-E_1)\lambda}\right) \tag{63}$$

$$\langle(\boldsymbol{r}\times(\boldsymbol{E}\times\boldsymbol{B}))_i\rangle_{\text{wave}}$$
$$=\frac{D}{r^2}\Bigg[\mathrm{i}\varepsilon_{ilk}\hat{r}^l\hat{j}^k\cos\theta(h(\lambda)-h^*(\lambda)) - (\hat{j}^i - \hat{r}^i\cos\theta)$$
$$\times\left(h(\lambda)+h^*(\lambda)+\frac{1}{\pi(E_2-E_1)\lambda}\right)\Bigg]\frac{1}{\lambda^2}. \tag{64}$$

If we now suppose that λ is big in comparison with the wavelength λ_{21} (but still has to be much smaller than r) and if we use the expansion

$$\mathrm{E_i}[-(E_1-E_2)\lambda]\exp[\mathrm{i}(E_1-E_2)\lambda] \approx \frac{\mathrm{i}}{(E_1-E_2)\lambda} - \frac{1}{((E_1-E_2)\lambda)^2} + \ldots \tag{64}$$

we see that beside the exponential terms connected with the broadening of energy level $2p_{3/2}$ there are also certain 'non-causal' tails:

$$\tfrac{1}{2}\langle \boldsymbol{E}^2\rangle_{\text{wave}} \simeq (\quad) + \frac{A}{r^2}2\frac{1}{\lambda^4} \tag{66}$$

$$\tfrac{1}{2}\langle \boldsymbol{B}^2\rangle_{\text{wave}} \simeq (\quad) + \frac{A}{r^2}(1+\cos^2\theta)\frac{1}{\lambda^4} \tag{67}$$

$$\langle[\boldsymbol{r}\times(\boldsymbol{E}\times\boldsymbol{B})]_i\rangle_{\text{wave}} \simeq (\quad) + \frac{B}{r^2}2\cos\theta\,\varepsilon_{ilk}\hat{r}^l\hat{j}^k\frac{1}{\lambda^4}. \tag{68}$$

The fact of the existence of such 'tails' is related to the excitation of the atom by one photon. One photon (or even any definite number of photons) cannot lead to a state which would give causal evolution for all operators (we mean here particularly operators bilinear in fields). The same tails appear in any definition of the excited state if we assume the excitation to be accomplished by one photon.

5. Summary

In this work we have considered the properties of the electromagnetic field around the hydrogen atom in the excited $2p_{3/2}$ state. The first problem dealt with was the definition of the excited state in quantum field theory. On the basis of the properties of the full electron propagator in the external Coulomb potential we have postulated, in section 2 a certain definition of this state. This definition (formula (6)) removes the unpleasant features of the 'bare' excited state like, for instance, surface terms. This definition has served in the following sections to find the energy density and the angular momentum density in the space around the atom. In the case of the excited state these quantities are built of two components: the virtual cloud contribution and the real wave contribution. For the virtual cloud we have got in section 3 the spatial distribution analogous to that of the ground state. Because of different geometry of 2p and 1s states the virtual cloud changes during the process of absorption and emission of the real photon. In the far (radiation) zone their dependence is the same as for the ground state: $1/r^7$.

The energy density and the angular momentum density associated with the real wave have been found in section 4. It was not possible to find them on the light cone because of infinities. Out of it the behaviour of these quantities is expressed through ordinary and integral exponent functions (formulae (58)-(60)). Far from the light cone, we have found, for all the quantities, tails of the type $1/\lambda^4$, where λ is the distance from the observation point to the maximum of wave position. They are connected with the formation of states of one photon.

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I would like very much to thank Professor Iwo Białynicki-Birula for the help during preparation of this work and Professor Francesco Persico for the interesting discussions and warm-hearted reception during my stays in Palermo.

Appendix 1. Some calculational methods used in this work

In this appendix we would like to calculate the wavepackets that show the manner in which state $|2'\rangle$ is decomposed into the one-photon in and out states. We would like also to explain the method of approximation used in this work in the place of the ordinary perturbation expansion.

The wavepackets in question are useful in sections 3 and 4 and in appendix 3. We do not give the decomposition into the more-than-one-photon states as we will not use it. Having used the reduction formulae (for instance Bjorken and Drell 1965, Itzykson and Zuber 1978), we find

$$\begin{aligned}\langle 1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{out}|2'\rangle &= \int \mathrm{d}^3x \langle 1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{out}|\bar{\Psi}(\boldsymbol{x}, 0)|0\rangle \gamma^0 \Psi_2(\boldsymbol{x}, 0) \\ &= -\int \mathrm{d}^4x_1\, \mathrm{d}^4x_2\, \mathrm{d}^3x\, \mathrm{e}^{\mathrm{i}kx_1} \varepsilon_{\boldsymbol{k}}^{\alpha(\lambda)} \partial_\sigma \partial^\sigma_{x_1} \bar{\Psi}_1(x_2) \mathrm{D}_v(x_2) \\ &\quad \times \langle 0|\mathrm{T}(\Psi(x_2) A_\alpha(x_1) \bar{\Psi}(\boldsymbol{x}, 0))|0\rangle \gamma^0 \Psi_2(\boldsymbol{x}, 0) \qquad \text{(A1.1)}\end{aligned}$$

where D_v denotes the Dirac operator in the Coulomb potential. To calculate the three-point Green function, which we need now, we cannot make use of the ordinary perturbation method, since, as was already mentioned, we would lose $i\Gamma/2$ in denominators. What we can take advantage of, instead, are Dyson-Schwinger equations (Dyson 1949, Schwinger 1951). The equation for the vertex function, written diagrammatically, is shown on figure 4.

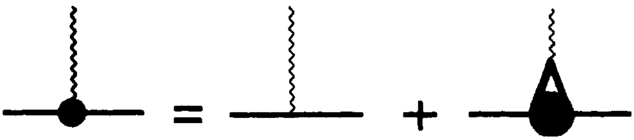

Figure 4. The Dyson-Schwinger equation for the vertex function.

The approximation that we use in the place of the standard perturbation expansion consists in retaining only the first term. This means that

$$\langle 0|\mathrm{T}(\Psi(x_2)A_\alpha(x_1)\bar{\Psi}(\boldsymbol{x},0))|0\rangle \simeq -e\int \mathrm{d}^4w\, S_v^{\mathrm{F}}(x_2,w)\gamma^\mu S_v^{\mathrm{F}}(w,\boldsymbol{x},0)\mathrm{D}^{\mathrm{F}}_{\mu\alpha}(x_1-w) \tag{A1.2}$$

where S_v^{F} is the *full, renormalized* electron propagator and $D^{\mathrm{F}}_{\mu\alpha}$ (in our gauge it is equal to $g_{\mu\alpha}\Delta^{\mathrm{F}}$) is the *free* photon propagator. The same type of approximation was applied in section 3 for the higher Green functions. The behaviour of the full propagator S_v^{F} causes the appropriate denominators to now have the 'good' form: $E-E_2\pm i\Gamma/2$.

Inserting (A1.2) into (A1.1), we come to

$$\langle 1s,(\boldsymbol{k},\lambda)\mathrm{out}|2'\rangle = e\int \mathrm{d}^3x\, \mathrm{e}^{-\mathrm{i}\boldsymbol{kx}}\varepsilon_{\boldsymbol{k}}^{\alpha(\lambda)}\bar{\Psi}_1(\boldsymbol{x})\gamma^\alpha\Psi_2(\boldsymbol{x})\frac{1}{k+E_1-E_2+\mathrm{i}\Gamma/2} \tag{A1.3}$$

after having neglected the terms with higher powers of e. In an analogous way we get

$$\langle 1s,(\boldsymbol{k},\lambda)\mathrm{in}|2'\rangle = e\int \mathrm{d}^3x\, \mathrm{e}^{-\mathrm{i}\boldsymbol{kx}}\varepsilon_{\boldsymbol{k}}^{\alpha(\lambda)}\bar{\Psi}_1(\boldsymbol{x})\gamma^\alpha\Psi_2(\boldsymbol{x})\frac{1}{k+E_1-E_2-\mathrm{i}\Gamma/2}. \tag{A1.4}$$

Appendix 2. Various formulae concerning excited state

$$\sum_{\boldsymbol{q},\rho}\!\!\!\!\int \langle 2'|1s,(\boldsymbol{q},\rho)\mathrm{in}\rangle\langle 1s,(\boldsymbol{q},\rho)\mathrm{in}|1s,(\boldsymbol{k}\lambda)\mathrm{out}\rangle$$
$$= \langle 2'|1s,(\boldsymbol{k}\lambda)\mathrm{out}\rangle + \text{higher-order terms} \tag{A2.1}$$

$$\sum_{\boldsymbol{k},\lambda}\!\!\!\!\int e^2\frac{\int \mathrm{d}^3u\,\mathrm{e}^{\mathrm{i}\boldsymbol{ku}}\varepsilon_{\boldsymbol{k}}^{\alpha(\lambda)*}\bar{\Psi}_2(\boldsymbol{u})\gamma_\alpha\Psi_1(\boldsymbol{u})\int \mathrm{d}^3w\,\mathrm{e}^{-\mathrm{i}\boldsymbol{kw}}\varepsilon_{\boldsymbol{k}}^{\beta(\lambda)}\bar{\Psi}_1(\boldsymbol{w})\gamma_\beta\Psi_2(\boldsymbol{w})}{(k+E_1-E_2-\mathrm{i}\Gamma/2)(k+E_1-E_2+\mathrm{i}\Gamma/2)}\exp[\mathrm{i}(k+E_1)\tau]$$
$$= \Theta(\tau)\exp[\mathrm{i}(E_2+\mathrm{i}\Gamma/2)\tau] + \Theta(-\tau)\exp[\mathrm{i}(E_2-\mathrm{i}\Gamma/2)\tau] + O\left(\frac{\Gamma}{\Delta E}\right) \tag{A2.2}$$

$$\int_0^\infty \mathrm{d}\omega\,\mathrm{e}^{\mathrm{i}\omega\tau}\left(\frac{1}{\omega+\Delta E+\mathrm{i}\Gamma/2}-\frac{1}{\omega+\Delta E-\mathrm{i}\Gamma/2}\right)$$
$$= -\mathrm{i}\Gamma\int_0^\infty \mathrm{d}\omega\,\mathrm{e}^{\mathrm{i}\omega\tau}\frac{1}{(\omega+\Delta E)^2+\Gamma^2/4}. \tag{A2.3}$$

But

$$\left|\Gamma\int_0^\infty d\omega\, e^{i\omega\tau}\frac{1}{(\omega+\Delta E)^2+\Gamma^2/4}\right|$$
$$\leqslant \Gamma\int_0^\infty d\omega\frac{1}{(\omega+\Delta E)^2+\Gamma^2/4}$$
$$=2\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{\Gamma}{2\Delta E}\right)\right]\approx\frac{\Gamma}{\Delta E}. \tag{A2.4}$$

In similar way

$$\int_0^\infty d\omega\, e^{i\omega\tau}\left(\frac{1}{\omega+\Delta E+i\Gamma}-\frac{1}{\omega+\Delta E+i\varepsilon}\right)\approx O\left(\frac{\Gamma}{\Delta E}\right) \tag{A2.5}$$

$$\int d^4x\,\Psi_2^+(\boldsymbol{x})S_v^F(x,y)\exp[i(k+E_1)x_0]$$
$$\simeq\frac{1}{k+E_1-E_2+i\Gamma/2}\bar{\Psi}_2(y)\exp[i(k+E_1)y_0] \tag{A2.6}$$

$$\int d^4x\,S_v^F(y,x)\gamma^0\Psi_2(\boldsymbol{x})\exp[-i(k+E_1)x_0]$$
$$\simeq\frac{1}{k+E_1-E_2+i\Gamma/2}\Psi_2(y)\exp[-i(k+E_1)y_0]. \tag{A2.7}$$

Appendix 3. The properties of the state $|2'\rangle$

Here we would like to show what happens when instead of the state $|\mathbf{2}\rangle$ we use $|2'\rangle$. Some features of $|2'\rangle$ become visible already in the simple case, linear in A—the magnetic field calculation. Let us then find

$$I^{\mu\prime}(x)=\langle 2'|\mathscr{A}^\mu(x)|2'\rangle=\langle 2'|A^\mu(x)|2'\rangle+A^\mu_{cl}(x) \tag{A3.1}$$

where $\mathscr{A}^\mu$ is the total field, A^μ_{cl} the classical part of it (proton) and A^μ the quantum part.

Suppose for a moment that $x_0>0$. If so, let us insert on the left of the field operator A^μ a complete set of out states (we now consider only the quantum part of $\mathscr{A}$—the classical part does not give any contribution to the magnetic field).

$$I_1^{\mu\prime}(x)=\sum_n\!\!\!\!\!\!\int\int d^3u\,d^3w\,\Psi_2^+(\boldsymbol{u},0)\langle 0|\Psi(\boldsymbol{u},0)|n\,\text{out}\rangle$$
$$\times\langle n\,\text{out}|T(A^\mu(x)\bar{\Psi}(\boldsymbol{w},0))|0\rangle\gamma^0\Psi_2(\boldsymbol{w},0). \tag{A3.2}$$

We want to calculate this expression in the lowest order of perturbation theory, i.e. we expect only one e to stand before everything, e which is connected with the joining of A to the electron line (this remark does not concern the widths Γ also present in this expression, which contain e as well, and which have to be treated 'non-perturbatively' since they go together with time). In the order in question only one- or two-photon out states are involved (plus the atom in the ground state): $|1s,(\boldsymbol{k},\lambda)\text{out}\rangle$ and $|1s,(\boldsymbol{k},\lambda),(\boldsymbol{q},\rho)\text{out}\rangle$ as illustrated in figure 5.

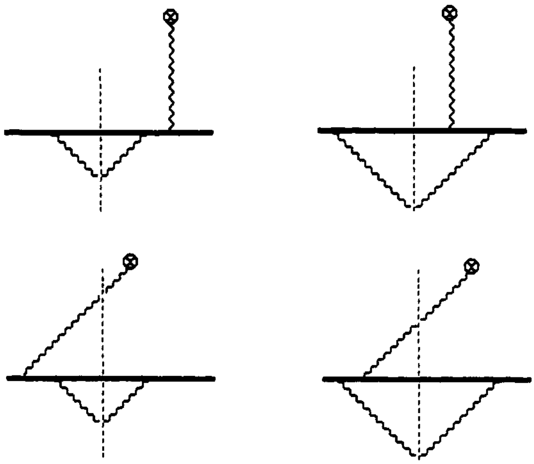

Figure 5. The diagrams that contribute to (A3.2) in the lowest order in e. The broken vertical line expresses the sum over out states. This means that the diagram lying to the left corresponds to complex conjugate functions.

The admission of a greater number of out photons leads to expressions with higher powers of e. The diagrams of the figure 5 are of order e and not e^3 due to the *resonant* photon exchanged between the left and the right part of the diagrams. The fact that the photon has to be resonant to give a diagram of the appropriate order of the perturbation expansion, exerts strong influence upon the manner of calculations. We will see it clearly while calculating $\langle \mathbf{2}|A^\mu(x)|\mathbf{2}\rangle$—the expression (A3.12) and those following it.

$$\begin{aligned}
I_1^{\mu\prime}(x) = \sum_{k,\lambda}\!\!\!\!\!\!\int \int & \mathrm{d}^3u\, \mathrm{d}^3w\, \Psi_2^+(\boldsymbol{u},0)\langle 0|\Psi(\boldsymbol{u},0)|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{out}\rangle \\
& \langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{out}|\mathrm{T}(A^\mu(x)\bar{\Psi}(\boldsymbol{w},0))|0\rangle\gamma^0\Psi_2(\boldsymbol{w},0) \\
& + \sum_{k,\lambda}\!\!\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\!\!\int \int \mathrm{d}^3u\, \mathrm{d}^3w\, \Psi_2^+(\boldsymbol{u},0)\langle 0|\Psi(\boldsymbol{u},0)|1\mathrm{s},(\boldsymbol{k},\lambda),(\boldsymbol{q},\rho)\mathrm{out}\rangle \\
& \times\tfrac{1}{2}\langle 1\mathrm{s},(\boldsymbol{k},\lambda),(\boldsymbol{q},\rho)\mathrm{out}|\mathrm{T}(A^\mu(x)\bar{\Psi}(\boldsymbol{w},0))|0\rangle\gamma^0\Psi_2(\boldsymbol{w},0) \\
= \sum_{k,\lambda}\!\!\!\!\!\!\int \int & \mathrm{d}^3u\, \mathrm{d}^3w\, \Psi_2^+(\boldsymbol{u},0)\langle 0|\Psi(\boldsymbol{u},0)|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{out}\rangle \\
& \times\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{out}|\mathrm{T}(A^\mu(x)\bar{\Psi}(\boldsymbol{w},0))|0\rangle\gamma^0\Psi_2(\boldsymbol{w},0) \\
& + \sum_{k,\lambda}\!\!\!\!\!\!\int \int \mathrm{d}^3u\, \mathrm{d}^3w \int \mathrm{d}^4z\, \Delta^-(x-z)\partial_\alpha\partial_z^\alpha\Psi_2^+(\boldsymbol{u},0) \\
& \times\langle 0|\tilde{\mathrm{T}}(\Psi(\boldsymbol{u},0)A^\mu(z))|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{out}\rangle \\
& \times\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{out}|\bar{\Psi}(\boldsymbol{w},0)|0\rangle\gamma^0\Psi_2(\boldsymbol{w},0)
\end{aligned} \qquad (\mathrm{A3.3})$$

where $\tilde{\mathrm{T}}$ denotes the operator of antichronological ordering. After making use of the reduction formulae and the Dyson–Schwinger equations in the way described in appendix 1, we get

$$I_1^{\mu'}(x)=\sum_{k,\lambda}\!\!\!\!\int \int \mathrm{d}^3u\,\mathrm{d}^3v\,\varepsilon_k^{\alpha^*(\lambda)}\bar{\Psi}_2(\boldsymbol{u})\gamma_\alpha\Psi_1(\boldsymbol{u})\frac{1}{k+E_1-E_2-\mathrm{i}\Gamma/2}\int \mathrm{d}^4w\,\mathrm{d}^4z$$
$$\times(\bar{\Psi}_1(w)\gamma_\beta\varepsilon_k^{\beta(\lambda)}\,\mathrm{e}^{\mathrm{i}kw}S_v^{\mathrm{F}}(w,z)\gamma^\mu S_v^{\mathrm{F}}(z,\boldsymbol{v},0)\gamma^0\Psi_2(\boldsymbol{v},0)\Delta^{\mathrm{F}}(x-z)$$
$$+\bar{\Psi}_1(w)\gamma^\mu S_v^{\mathrm{F}}(w,z)\gamma_\beta\varepsilon_k^{\beta(\lambda)}\,\mathrm{e}^{\mathrm{i}kz}S_v^{\mathrm{F}}(z,\boldsymbol{v},0)\gamma^0\Psi_2(\boldsymbol{v},0)\Delta^{\mathrm{F}}(x-w))$$
$$=e\int \mathrm{d}^4w\,\Theta(w_0)\,\mathrm{e}^{-\Gamma w_0}\bar{\Psi}_2(w)\gamma^\mu\Psi_2(w)\Delta^{\mathrm{F}}(x-w)$$
$$+e\int \mathrm{d}^4w\,\Theta(w_0)(1-\mathrm{e}^{-\Gamma w_0})\bar{\Psi}_1(w)\gamma^\mu\Psi_1(w)\Delta^{\mathrm{F}}(x-w) \tag{A3.4}$$

where we have additionally used the formulae (A2.2), (A2.6) and (A2.7). The whole calculation is presented here with significant abbreviation. A more complete one will be shown in the context of state $|\mathbf{2}\rangle$. The second term of (A3.3) can be obtained from (A3.4) by complex conjugation and by replacing the antiFeynman propagator $\Delta^{\bar{\mathrm{F}}}$ with Δ^-. If we also take into account that

$$\Delta^{\mathrm{F}}(x)+\Delta^-(x)=\Delta^{\mathrm{R}}(x)=\frac{1}{4\pi|\boldsymbol{x}|}\,\delta(x_0-|\boldsymbol{x}|) \tag{A3.5}$$

we will get for $I_1^{\mu'}$:

$$I_1^{\mu'}(x)=\frac{e}{4\pi}\int \mathrm{d}^3w\,\bar{\Psi}_2(\boldsymbol{w})\gamma^\mu\Psi_2(\boldsymbol{w})\frac{1}{|\boldsymbol{x}-\boldsymbol{w}|}\,\Theta(x_0-|\boldsymbol{x}-\boldsymbol{w}|)\exp[-\Gamma(x_0-|\boldsymbol{x}-\boldsymbol{w}|)]$$
$$+\frac{e}{4\pi}\int \mathrm{d}^3w\,\bar{\Psi}_1(\boldsymbol{w})\gamma^\mu\Psi_1(\boldsymbol{w})\frac{1}{|\boldsymbol{x}-\boldsymbol{w}|}\,\Theta(x_0-|\boldsymbol{x}-\boldsymbol{w}|)$$
$$\times\{1-\exp[-\Gamma(x_0-|\boldsymbol{x}-\boldsymbol{w}|)]\}. \tag{A3.6}$$

We have assumed above that $x_0>0$. For our purposes it is, however, not essential to consider the case $x_0<0$. Whatever this second term would be, it certainly gives no contribution for $t\in(0,r/c)$. We see then that we do not 'measure' anything until $t=r/c$. This is connected with an unsatisfactory definition of the state $|2'\rangle$. If we now calculate the magnetic field for which the performing of curl is needed, the surface terms of the type δ, spoken of in section 2, will emerge. It is a consequence of the turning on the interaction at $t=0$. We will not take this calculation further; our aim was only to show that the state $|2'\rangle$ is not a physical one. Now we will see, instead, that for $|\mathbf{2}\rangle$ defined in (6), the above drawbacks disappear:

$$I^\mu(x)=\langle\mathbf{2}|\mathscr{A}^\mu(x)|\mathbf{2}\rangle$$
$$=\sum_{k,\lambda}\!\!\!\!\int\sum_{q,\rho}\!\!\!\!\int\langle 2'|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|A^\mu(x)|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle$$
$$\times\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|2'\rangle+A^\mu_{\mathrm{cl}}(x). \tag{A3.7}$$

To get the desired transition amplitude one ought to put to the left of $A^\mu(x)$ a complete set of the out states:

$$I_1^\mu(x)=\sum_{k,\lambda}\!\!\!\!\int\sum_{q,\rho}\!\!\!\!\int\sum_{n}\!\!\!\int\langle 2'|1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{k},\lambda)\mathrm{in}|n\mathrm{out}\rangle$$
$$\times\langle n\mathrm{out}|A^\mu(x)|1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}\rangle\langle 1\mathrm{s},(\boldsymbol{q},\rho)\mathrm{in}|2'\rangle. \tag{A3.8}$$

This time our expression is more complicated than (A3.2)—there are more combinations of different photons: in, out and A^μ, despite the fact that, as before, only one- and two-photon out states are involved in the lowest order in e. The three sums over the states give rise to many diagrams (figures 6 and 7).

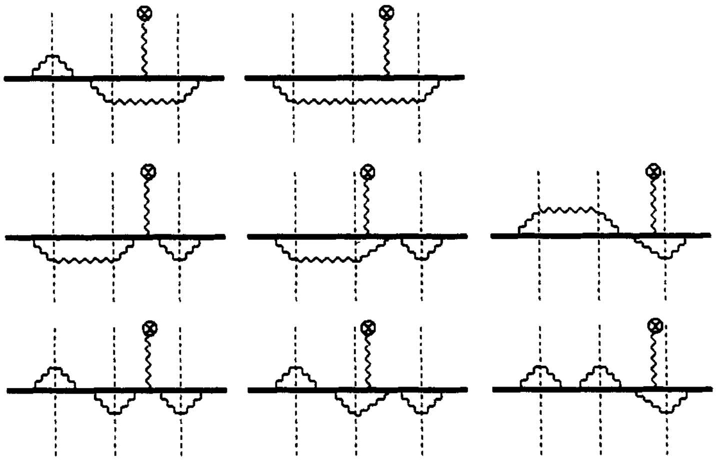

Figure 6. The diagrams contributing to (A3.8) if $|n\text{out}\rangle$ is a one-photon state.

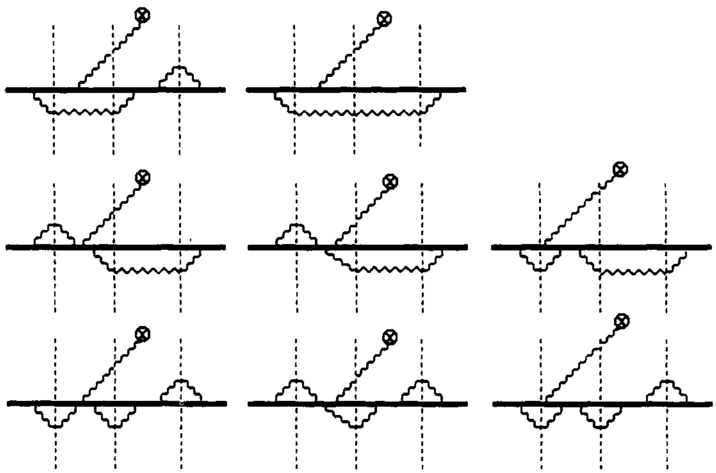

Figure 7. The diagrams contributing to (A3.8) if $|n\text{out}\rangle$ is a two-photon state.

Let us first evaluate the expression $I^\mu_{1a}(x)$ which corresponds to the one-photon out states. We start by calculating the object

$$
\begin{aligned}
&\langle 1\mathrm{s}, (\boldsymbol{p}, \sigma)\mathrm{out}|A^\mu(x)|1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in}\rangle \\
&\qquad = \int \mathrm{d}^4x_1\, \mathrm{d}^4x_2\, \mathrm{d}^4y_1\, \mathrm{d}^4y_2\, \mathrm{e}^{\mathrm{i}px_1} \varepsilon_{\boldsymbol{p}}^{\alpha(\sigma)} \partial_\rho \partial^\rho_{x_1} \bar{\Psi}_1(x_2) \mathrm{D}_\upsilon(x_2) \\
&\qquad\quad \times \langle 0|\mathrm{T}(\Psi(x_2) A_\alpha(x_1) A^\mu(x) A_\beta(y_1) \bar{\Psi}(y_2))|0\rangle \overleftarrow{\mathrm{D}}_\upsilon(y_2)^* \\
&\qquad\quad \times \Psi_1(y_2) \partial_\rho \overleftarrow{\partial^\rho_{y_1}}\, \mathrm{e}^{-\mathrm{i}ky_1} \varepsilon_{\boldsymbol{k}}^{\beta(\lambda)*} + \delta_\Gamma(\boldsymbol{p}-\boldsymbol{k}) \delta_{\lambda\sigma} \langle 1\mathrm{s}, 0\mathrm{out}|A^\mu(x)|1\mathrm{s}, 0\mathrm{in}\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= e^3 \int \mathrm{d}^4 w_1\, \mathrm{d}^4 w_2\, \mathrm{d}^4 w_3 [\bar{\Psi}_1(w_1)\, \mathrm{e}^{\mathrm{i}pw_1} (\varepsilon_p^\sigma \gamma)^\downarrow \\
&\times S_v^{\mathrm{F}}(w_1, w_2) \gamma^\mu S_v^{\mathrm{F}}(w_2, w_3)^\downarrow\, \mathrm{e}^{-\mathrm{i}kw_3} (\varepsilon_k^{(\lambda)*} \gamma) \Psi_1(w_3) \Delta^{\mathrm{F}}(x - w_2) \\
&+ \bar{\Psi}_1(w_1)\, \mathrm{e}^{\mathrm{i}pw_1} (\varepsilon_p^\sigma \gamma)^\downarrow S_v^{\mathrm{F}}(w_1, w_2)\, \mathrm{e}^{-\mathrm{i}kw_2} (\varepsilon_k^{(\lambda)*} \gamma) \\
&\times S_v^{\mathrm{F}}(w_2, w_3)^\downarrow \gamma^\mu \Psi_1(w_3) \Delta^{\mathrm{F}}(x - w_3) + \bar{\Psi}_1(w_1) \gamma^\mu S_v^{\mathrm{F}}(w_1, w_2)^\downarrow \\
&\times \mathrm{e}^{\mathrm{i}pw_2} (\varepsilon_p^\sigma \gamma) S_v^{\mathrm{F}}(w_2, w_3)^\downarrow\, \mathrm{e}^{-\mathrm{i}kw_3} (\varepsilon_k^{(\lambda)*} \gamma) \Psi_1(w_3) \Delta^{\mathrm{F}}(x - w_1)] \\
&+ \delta_\Gamma(\boldsymbol{p} - \boldsymbol{k}) \delta_{\lambda\sigma} \langle 1\mathrm{s}, 0\mathrm{out} | A^\mu(x) | 1\mathrm{s}, 0\mathrm{in} \rangle
\end{aligned} \tag{A3.9}
$$

where

$$
\delta_\Gamma(\boldsymbol{p} - \boldsymbol{k}) = (2\pi)^3 2 |\boldsymbol{k}| \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}). \tag{A3.10}
$$

In the places marked with arrows we now insert the unit operators expressed in the following way:

$$
\delta^{(3)}(\boldsymbol{x} - \boldsymbol{y}) = \sum_{n\pm} \Psi_n(\boldsymbol{x}) \Psi_n^+(\boldsymbol{y}) \tag{A3.11}
$$

where the Ψ_n are the Dirac atomic wavefur:ctions.

All the internal photons in the diagrams in figures 6 and 7 have to be associated with the transitions 2p↔1s; they have ot be the resonant photons. Only then, thanks to the confluence of the denominators, may the surplus of e in the numerators be cancelled. The appropriate poles arise only when n equals 2 or 1 in the sum (A3.11). Using formulae (A2.6) and (A2.7) and retaining only the resonance terms, we come to

$$
\begin{aligned}
&\langle 1\mathrm{s}, (\boldsymbol{p}, \sigma)\mathrm{out} | A^\mu(x) | 1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in} \rangle \\
&= e^3 \bigg(\int \mathrm{d}^3 w_1\, \mathrm{d}^4 w_2\, \mathrm{d}^3 w_3\, \bar{\Psi}_1(\boldsymbol{w}_1)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\boldsymbol{w}_1} (\varepsilon_p^{(\sigma)} \gamma) \Psi_2(\boldsymbol{w}_1) \\
&\times \frac{1}{p + E_1 - E_2 + \mathrm{i}\Gamma/2} \bar{\Psi}_2(\boldsymbol{w}_2) \gamma^\mu \Psi_2(\boldsymbol{w}_2) \exp[-\mathrm{i}(k + E_1) w_2^0] \\
&\times \bar{\Psi}_2(\boldsymbol{w}_3)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{w}_3} (\varepsilon_k^{(\lambda)*} \gamma) \Psi_1(\boldsymbol{w}_3) \frac{1}{k + E_1 - E_2 + \mathrm{i}\Gamma/2} \Delta^{\mathrm{F}}(x - w_2) \\
&- \mathrm{i} \int \mathrm{d}^3 w_1\, \mathrm{d}^4 w_2\, \mathrm{d}^4 w_3\, \bar{\Psi}_1(\boldsymbol{w}_1)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\boldsymbol{w}_1} (\varepsilon_p^{(\sigma)} \gamma) \Psi_2(\boldsymbol{w}_1) \\
&\times \frac{1}{p + E_1 - E_2 + \mathrm{i}\Gamma/2} \exp[\mathrm{i}(p + E_1 - k - E_1) w_2^0] \Theta(w_2^0 - w_3^0) \\
&\times \bar{\Psi}_2(\boldsymbol{w}_2)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{w}_2} (\varepsilon_k^{(\lambda)*} \gamma) \Psi_1(\boldsymbol{w}_2) \bar{\Psi}_1(\boldsymbol{w}_3) \gamma^\mu \Psi_1(\boldsymbol{w}_3) \Delta^{\mathrm{F}}(x - w_3) \\
&- \mathrm{i} \int \mathrm{d}^4 w_1\, \mathrm{d}^4 w_2\, \mathrm{d}^3 w_3\, \bar{\Psi}_1(\boldsymbol{w}_1) \gamma^\mu \Psi_1(\boldsymbol{w}_1) \Theta(w_1^0 - w_2^0) \\
&\times \bar{\Psi}_1(\boldsymbol{w}_2)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\boldsymbol{w}_2} (\varepsilon_p^{(\sigma)} \gamma) \Psi_2(\boldsymbol{w}_2) \exp[\mathrm{i}(p + E_1 - k - E_1) w_2^0] \\
&\times \bar{\Psi}_2(\boldsymbol{w}_3)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{w}_3} (\varepsilon_k^{(\lambda)*} \gamma) \Psi_1(\boldsymbol{w}_3) \frac{1}{k + E_1 - E_2 + \mathrm{i}\Gamma/2} \Delta^{\mathrm{F}}(x - w_1) \bigg) \\
&+ \delta_\Gamma(\boldsymbol{p} - \boldsymbol{k}) \delta_{\lambda\sigma} \langle 1\mathrm{s}, 0\mathrm{out} | A^\mu(x) | 1\mathrm{s}, 0\mathrm{in} \rangle.
\end{aligned} \tag{A3.12}
$$

To find $I^{\mu}_{1a}(x)$ we must now integrate (A3.9) with the wavepackets (A1.3) and (A1.4) after having earlier used (A2.1). There emerge sums of the kind (A2.2) and after having performed certain integrals and once again sums (A2.2), we finally come to the formula

$$\begin{aligned} I^{\mu}_{1a}(x) = e & \int d^4w(\Theta(w_0)\, e^{-\Gamma w_0} + \Theta(-w_0)\, e^{\Gamma w_0})\bar{\Psi}_2(w)\gamma^{\mu}\Psi_2(w)\Delta^{F}(x-w) \\ & + e \int d^4w\, \Theta(-w_0)(1 - e^{\Gamma w_0})\bar{\Psi}_1(w)\gamma^{\mu}\Psi_1(w)\Delta^{F}(x-w) \\ & + e \int d^4w\, \Theta(w_0)(1 - e^{-\Gamma w_0})\bar{\Psi}_1(w)\gamma^{\mu}\Psi_1(w)\Delta^{F}(x-w). \end{aligned} \tag{A3.13}$$

It is only a half of what we want to find—$I^{\mu}_{1b}(x)$ has now to be calculated. One can, however, facilitate the job by expressing $I^{\mu}_{1b}(x)$ through $I^{\mu}_{1a}(x)$, which has already been found:

$$\begin{aligned} I^{\mu}_{1b}(x) = & \sum_{k,\lambda}\!\!\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\!\!\int \sum_{p,\sigma}\!\!\!\!\!\!\int \sum_{l,\tau}\!\!\!\!\!\!\int \langle 2'|1\mathrm{s}, (\boldsymbol{q}, \rho)\mathrm{in}\rangle\langle 1\mathrm{s}, (\boldsymbol{q}, \rho)\mathrm{in}|1\mathrm{s}, (\boldsymbol{p}, \sigma), (\boldsymbol{l}, \tau)\mathrm{out}\rangle \\ & \times \tfrac{1}{2}\langle 1\mathrm{s}, (\boldsymbol{p}, \sigma), (\boldsymbol{l}, \tau)\mathrm{out}|A^{\mu}(x)|1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in}\rangle\langle 1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in}|2'\rangle \\ = & \int d^4z\, \Delta^{-}(x-z)\partial_{\rho}\partial^{\rho}_{z} \sum_{k,\lambda}\!\!\!\!\!\!\int \sum_{q,\rho}\!\!\!\!\!\!\int \sum_{p,\sigma}\!\!\!\!\!\!\int \langle 2'|1\mathrm{s}, (\boldsymbol{q}, \rho)\mathrm{in}\rangle \\ & \times \langle 1\mathrm{s}, (\boldsymbol{q}, \rho)\mathrm{in}|A^{\mu}(z)|1\mathrm{s}, (\boldsymbol{p}, \sigma)\mathrm{out}\rangle\langle 1\mathrm{s}, (\boldsymbol{p}, \sigma)\mathrm{out}|1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in}\rangle \\ & \times \langle 1\mathrm{s}, (\boldsymbol{k}, \lambda)\mathrm{in}|2'\rangle \end{aligned} \tag{A3.14}$$

or equivalently

$$I^{\mu}_{1b}(x) = \int d^4z\, \Delta^{-}(x-z)\partial_{\rho}\partial^{\rho}_{z} I^{\mu *}_{1a}(z). \tag{A3.15}$$

Now we take both pieces together: $I^{\mu}_{1} = I^{\mu}_{1a} + I^{\mu}_{1b}$:

$$\begin{aligned} I^{\mu}_{1}(x) = e & \int d^4w(\Theta(w_0)\, e^{-\Gamma w_0} + \Theta(-w_0)\, e^{\Gamma w_0})\bar{\Psi}_2(w)\gamma^{\mu}\Psi_2(w)\Delta^{R}(x-w) \\ & + e \int d^4w[\Theta(-w_0)(1 - e^{\Gamma w_0}) + \Theta(w_0)(1 - e^{-\Gamma w_0})] \\ & \times \bar{\Psi}_1(w)\gamma^{\mu}\Psi_1(w)\Delta^{R}(x-w) \\ = & \frac{e}{4\pi}\int d^3w\, \bar{\Psi}_2(\boldsymbol{w})\gamma^{\mu}\Psi_2(\boldsymbol{w})\frac{1}{|\boldsymbol{x}-\boldsymbol{w}|}\{\Theta(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)\exp[-\Gamma(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)] \\ & + \Theta(|\boldsymbol{x}-\boldsymbol{w}| - x_0)\exp[\Gamma(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)]\} \\ & + \frac{e}{4\pi}\int d^3w\, \bar{\Psi}_1(\boldsymbol{w})\gamma^{\mu}\Psi_1(\boldsymbol{w})\frac{1}{|\boldsymbol{x}-\boldsymbol{w}|}[\![\Theta(x_0 - |\boldsymbol{x}-\boldsymbol{w}|) \\ & \times \{1 - \exp[-\Gamma(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)]\} + \Theta(|\boldsymbol{x}-\boldsymbol{w}| - x_0) \\ & \times \{1 - \exp[\Gamma(x_0 - |\boldsymbol{x}-\boldsymbol{w}|)]\}]\!]. \end{aligned} \tag{A3.16}$$

In contradistinction to (A3.6), this expression is valid now for all times. It is very nice to observe how both terms of (A3.16) come out automatically. It was sufficient to assume that the propagator S has a pole and the Feynman diagrams of the figures 6 and 7 give all we would expect: the excitation and the decay of the 2p state and the 1s state before and after excitation. As before, we got in (A3.16) the retarded propagators; the time evolution, however, now has no hole between $-r/c$ and r/c! This suggests that the definition of the excited state (6) actually works.

Having derived (A3.16) we can easily obtain the magnetic field. This time no δ type terms arise. As the wavefunctions Ψ_2 are known, we only have to perform the integrals over the source distribution, getting

$$B^k(\boldsymbol{x},t)=-\frac{1}{4\pi}\frac{e}{2m}\bigg(\frac{1+2\sqrt{4-(z\alpha)^2}}{5}\times\mathbf{2}\times(\Theta(t-x)\,\mathrm{e}^{-\Gamma(t-x)}+\Theta(x-t)\,\mathrm{e}^{\Gamma(t-x)})$$
$$+\frac{1+2\sqrt{1-(z\alpha)^2}}{3}\times 1\times[\Theta(t-x)(1-\mathrm{e}^{-\Gamma(t-x)})$$
$$+\Theta(x-t)(1-\mathrm{e}^{\Gamma(t-x)})]\bigg)(\hat{j}^k-3\hat{x}^k(\hat{x}\cdot\hat{j}))\frac{1}{x^3} \tag{A3.17}$$

where **2** in the first term comes from

$$L_z+2S_z=1+2\times\tfrac{1}{2}=\mathbf{2}. \tag{A3.18}$$

For the second term we have

$$L_z+2S_z=0+2\times\tfrac{1}{2}=\mathbf{1}. \tag{A3.19}$$

In (A3.17) $\hat{j}$ is a unit vector in the direction of the overall angular momentum.

Appendix 4. Different relations and representations for propagators Δ used in the work

$$\Delta^{\mathrm{F}}(x)=\frac{1}{8\pi^2|\boldsymbol{x}|}\int_0^\infty \mathrm{d}\omega\{\exp[\mathrm{i}\omega(|\boldsymbol{x}|-x_0+\mathrm{i}\varepsilon)]+\exp[\mathrm{i}\omega(|\boldsymbol{x}|+x_0+\mathrm{i}\varepsilon)]\} \tag{A4.1}$$

$$\Delta^{\bar{\mathrm{F}}}(x)=\frac{1}{8\pi^2|\boldsymbol{x}|}\int_0^\infty \mathrm{d}\omega\{\exp[\mathrm{i}\omega(x_0-|\boldsymbol{x}|+\mathrm{i}\varepsilon)]+\exp[\mathrm{i}\omega(-|\boldsymbol{x}|-x_0+\mathrm{i}\varepsilon)]\} \tag{A4.2}$$

$$\Delta^{-}(x)=\frac{1}{8\pi^2|\boldsymbol{x}|}\int_0^\infty \mathrm{d}\omega\{\exp[\mathrm{i}\omega(x_0-|\boldsymbol{x}|+\mathrm{i}\varepsilon)]-\exp[\mathrm{i}\omega(|\boldsymbol{x}|+x_0+\mathrm{i}\varepsilon])\} \tag{A4.3}$$

$$\Delta^{+}(x)=\frac{1}{8\pi^2|\boldsymbol{x}|}\int_0^\infty \mathrm{d}\omega\{\exp[\mathrm{i}\omega(|\boldsymbol{x}|-x_0|+\mathrm{i}\varepsilon)]-\exp[\mathrm{i}\omega(-|\boldsymbol{x}|-x_0+\mathrm{i}\varepsilon)]\} \tag{A4.4}$$

$$\Delta^{\mathrm{F}}(x)+\Delta^{-}(x)=\Delta^{\mathrm{R}}(x) \tag{A4.5}$$

$$\Delta^{\bar{\mathrm{F}}}(x)=\Delta^{\mathrm{A}}(x)+\Delta^{\mathrm{R}}(x)-\Delta^{\mathrm{F}}(x) \tag{A4.6}$$

$$\Delta^{+}(x)=-\Delta^{-}(x)-\Delta^{\mathrm{A}}(x)+\Delta^{\mathrm{R}}(x). \tag{A4.7}$$

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